SPHERICAL RADIATION-DRIVEN SHOCK WAVES
IN A NON-UNIFORM ATMOSPHERE

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Abstract. Similarity solutions are obtained for spherical radiation-driven shock waves propagating in a non-uniform atmosphere at rest obeying a density power law. Approximate analytical solutions are also obtained and found to be in good agreement with the numerical solutions. The effect of the parameter characterizing the initial density distribution of the gas on solutions of the flow field is studied in detail. It is also shown analytically that the shock wave propagates as an overdriven detonation.

1. Introduction

Self-similar solutions for spherical blast waves in a non-uniform atmosphere obeying a power law for the density were given by Sedov (1959) and Rogers (1957) with special reference to astrophysical applications. These authors have assumed that the total energy of the flow is constant. However, the total energy of the flow ceases to be a constant if we take into consideration the addition of energy at the wave front due to the incident radiation. Wilson and Turcotte (1970) studied the strong spherical shocks in a uniform medium when the radiation is propagating radially inwards with constant power. An extension of this problem in a non-uniform medium was considered by Singh and Lal Srivastava (1982) but not solved. These authors, however, obtained the solutions for the case of constant initial density which was solved completely by Wilson and Turcotte (1970). Moreover, the solutions of Singh and Lal Srivastava (1982) are not at all agreeing with those obtained by Wilson and Turcotte (1970).

In this paper we study the problem of self-similar flows behind radiation-driven shock waves with spherical symmetry propagating in a non-uniform atmosphere obeying a power law

\[ \rho_0(r) = Ar^{-w} \quad (w < 2), \]

where \( A \) and \( w \) are constants. It is also assumed that the radiation propagates radially inwards with a constant power \( P \) which is completely absorbed within the expanding shock layer (cf. Wilson and Turcotte, 1970). The flow behind the strong shock is taken to be particle isentropic. The phase plane analysis of Sedov (1959) and Wilson and Turcotte (1970) is used for obtaining numerical solutions to this problem. Approximate analytical solutions are also obtained by using the integral method developed by Laumbach and Probstein (1969) and are found to be in good agreement with numerical solutions. Negative values of \( w \) are also considered and found that approximate solutions are more closer to numerical solutions as \( w \)
decreases. It is proved analytically that the shock wave propagates as an overdriven detonation.

2. Numerical Solutions

The equations of motion for spherical flows behind strong shock waves propagating in an ideal gas at rest are written after simplifications (see Sedov, 1959; and Wilson and Turcotte, 1970) as

\[ \frac{dX}{dV} = \frac{X}{V-\delta} \frac{M(V, X)}{N(V, X)}, \]

\[ \frac{d\ln \lambda}{dV} = -\frac{D(V, X)}{N(V, X)}, \]

\[ \frac{d\ln R}{d\ln \lambda} = \frac{(3-w)V}{(\delta-V)} \frac{N(V, X)}{(\delta-V)D(V, X)}, \]

where

\[ D(V, X) = (\delta-V)^2 X - 1, \]

\[ N(V, X) = V(1-V)(\delta-V)X - 3(V-K), \]

\[ M(V, X) = (\gamma-1)N(V, X) + [2-(3\gamma-1)V]D(V, X), \]

\[ K = \frac{[2(1-\delta)+w\delta]}{3\gamma}. \]

To obtain the above equations, we have used the following similarity transformation:

\[ V(\lambda) = \frac{ut}{r}, \quad R(\lambda) = \frac{\rho}{\rho_0}, \quad X(\lambda) = \frac{\rho}{\gamma p} \left( \frac{t}{t} \right)^2, \]

\[ \lambda = \left( \frac{A}{P} \right)^{\delta/3} r t^{-\delta}, \quad \delta = \frac{3}{5-w}. \]

The strong shock conditions may be written as

\[ V_s = (1-\beta)\delta, \]

\[ R_s = 1/\beta, \]

\[ X_s = [\gamma\beta(1-\beta)\delta^2]^{-1}, \]

where \( \beta \) is the shock density ratio to be determined and suffix \( s \) represents quantities immediately behind the shock. The value of \( \lambda \) at the shock is taken as \( \lambda_s \), which is related to the shock position \( r_s \) by

\[ r_s = \lambda_s \left( \frac{P}{A} \right)^{\delta/3} t^\delta. \]