Abstract. An approximate form for the $H$-function for isotropic scattering developed by Karanjai and Sen (1971) has been extended to the case of four-term scattering indicatrix.

1. Introduction


The $H$-function is the solution of the nonlinear integral equation

$$\frac{1}{H(\mu, w)} = 1 - \mu \int_0^1 \frac{\psi(\mu')H(\mu', w)}{\mu + \mu'} \, d\mu'$$

for anisotropic scattering with the scattering indicatrix $\sum_{i=0}^{3} x_{i} P_{i}(\cos \theta)$, the $H$-function is defined in terms of characteristic function

$$\psi(\mu) = \frac{w}{2} \sum_{i=0}^{3} x_{i} R_{i}(\mu) P_{i}(\mu),$$

where $R_{i}(\mu)$s satisfy the recurrence relation

$$iR_{i}(\mu) + (i - 1)R_{i-2}(\mu) = (2i - 1 - wx_{i-1})\mu R_{i-1}(\mu)$$

and

$$R_{0}(\mu) = 1, \quad R_{1}(\mu) = (1 - w)\mu.$$
From relation (3) we get $R_2(\mu)$ and $R_3(\mu)$ as
\begin{align*}
R_2(\mu) &= \frac{1}{2} \{(1 - w)(3 - wx_1)\mu^2 - 1\}, \\
R_3(\mu) &= \frac{1}{2}(5 - wx_2)\frac{1}{2}\mu \{(1 - w)(3 - wx_1)\mu^2 - 1\} - \frac{2}{3}(1 - w)\mu; \\
\end{align*}
where $P_l(\mu)$s being the Legendre polynomials
\begin{align*}
P_0(\mu) &= 1, \quad P_1(\mu) = \mu, \quad P_2(\mu) = \frac{1}{2}(3\mu^2 - 1)
\end{align*}
and
\begin{align*}
P_3(\mu) &= \frac{1}{2}(5\mu^3 - 3\mu).
\end{align*}
The $H$-function satisfies the equations:
\begin{align*}
(1) & \quad \int_0^1 H(\mu) \psi(\mu) \, d\mu = 1 - \left[ 1 - 2 \int_0^1 \psi(\mu) \, d\mu \right]^{1/2}, \\
(2) & \quad \left[ 1 - 2 \int_0^1 \psi(\mu) \, d\mu \right]^{1/2} \int_0^1 H(\mu) \psi(\mu) \mu^2 \, d\mu + \\
& \quad + \frac{1}{2} \left[ \int_0^1 H(\mu) \psi(\mu) \, d\mu \right]^2 = \int_0^1 \psi(\mu) \mu^2 \, d\mu, \\
(3) & \quad \int_0^1 \frac{H(\mu) \psi(\mu)}{1 - k\mu} = 1.
\end{align*}
The term $k$ in Equation (10), which is a function of $w, x_1, x_2,$ and $x_3$ satisfies the following transcendental equation (cf. Kolesov and Smoktii, 1972) as
\begin{align*}
& \frac{w}{2k} \ln \frac{1 + k}{1 - k} \left\{ 1 + \frac{x_1}{k^2} (1 - w) + \frac{x_2}{4k^4} (3 - k^2) [(3 - wx_1)(1 - w) - k^2] + \\
& \quad + \frac{wx_3}{24k^7} (5 - 3k^2) [(3 - wx_1)(1 - w) - k^2] (5 - wx_2) - \\
& \quad - \frac{wx_3}{6k^4} (5 - 3k^2)(1 - w) \right\} - \frac{wx_1}{k^2} (1 - w) - \\
& \quad - \frac{3wx_2}{4k^4} [(3 - wx_1)(1 - w) - k^2] + \frac{wx_3}{36k^6} (15 - 4k^2) \times \\
& \times \{ 4k^2(1 - w) - (5 - 3x_2) [(3 - wx_1)(1 - w) - k^2] \} = 1.
\end{align*}