CHARACTERISTIC ACTIONS $h^{(s)}$ IN THE STRUCTURE OF
THE UNIVERSE

The Angular Momentum–Mass Relation of Astronomical Systems

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Abstract. It is suggested that gravitationally bound systems in the Universe can be characterized by a set of actions $h^{(s)}$. The actions

$$h^{(s)} = \left( h \left( \frac{1}{2 \pi \frac{C^5}{GH_0^5}} \right)^{\frac{1}{2}} \left( \frac{1}{2 \pi \frac{C^5}{GH_0^5}} \right) \right),$$

derived from general theoretical consideration, are only determined by the fundamental physical constants (Planck's action $h$, the velocity of light $C$, gravitational constant $G$, and Hubble's constant $H_0$) and a scale parameter $s$. It is shown that $s = 1, 2, 3$ correspond, respectively, to the scales of galaxies, stars, and larger asteroids.

The spectra of the characteristic angular momenta and masses for gravitationally bound systems in the Universe are estimated by $J^{(s)} = h^{(s)}$ and $M^{(s)} = (h^{(s)}C/aG)^{1/2}$. Taken together, an angular momentum–mass relation is obtained, $J^{(s)} = A(M^{(s)})^2$, where $A = G/Ca$, $a = \frac{1}{137}$, for the astronomical systems observed on every scale. This $J–M$ relation is consistent with Brosche's empirical relation (Brosche, 1974).

1. Introduction

It is generally known that there are many gravitationally-bound astronomical systems in the Universe. The familiar systems are asteroids, planets, stars, star clusters, galaxies, clusters of galaxies, and superclusters. The basic dynamic variables of an astronomical system (mass, angular momentum, etc.) can be determined or estimated by observations. Two results inferred from observations are highly important for our discussions, which are follows:

(1) There exist some noticeable physical scales in the observed Universe. For example, if we take the distribution of the masses of observed astronomical objects into account, the orders $10^{34} \text{ g}, 10^{33} \text{ g}$, etc., of mass are typical scales because there are many typical objects – galaxies with a mass $M_g \sim 10^{34} \text{ g}$ (henceforth the symbol ‘~’ means ‘about the same order’) and stars with a mass $M_\ast \sim 10^{33} \text{ g}$. That is to say, a very significant fact is that some preferred scales are associated with different objects.

(2) As was pointed out by Brosche (1963), a wide variety of astronomical systems obey roughly a relation between their angular momenta $J$ and masses $M$, which is

$$J = AM^2.$$

(1)
The proportional factor $A$ in this empirical relation is about

$$A \sim 10^3 \frac{G}{C} \approx 2.2 \times 10^{-15} \text{ cm}^2 \text{ g}^{-1} \text{ s}^{-1},$$

(2)

where $G$ is the gravitational constant and $C$ is the velocity of light (Brosche, 1974). This is a very significant relation, because it implies that the behaviour of astronomical systems which are characterized by the basic dynamical variables (angular momentum $J$ and mass $M$) are probably dependent on each other.

One may ask, naturally, whence such phenomena about the structure of the Universe might arise. It is evident that these two observed results about the structure of the Universe should be consistent with the fundamental laws of physics. In this paper, we shall try to give them a united explanation from a very general theoretical consideration.

In Section 2 we shall give a set of the actions $h(s)$ by means of general theoretical analyses. The actions $h(s)$, which depend only on one scale parameter $s$ and the constants of nature which occur in the known fundamental laws of physics, seem to be fundamentally characteristic quantities of the gravitationally bound systems in the Universe. In Section 3, as the theoretical results suggest, we shall give the spectra of characteristic angular momenta and masses of the gravitationally bound systems $J(s)$ and $M(s)$, and the relation between them, $J(s) = A(M(s))^2$, where the factor $A = G/\nu C = (hC/e^2)G/C \approx 137 G/C$ is only determined by the fundamental constants. They coincide with observations. Conclusions and discussions are summarized in Section 4.

2. Characteristic Actions $h^{(s)}$ in the Structure of the Universe

The two facts mentioned in Section 1, taken together, mean that the behaviour of gravitationally bound systems characterized by the basic variables (say, angular momentum $J$, mass $M$, etc.) may be interrelated on different scales in the Universe. This can be stated as follows: the behaviour of gravitationally bound systems on a local scale is determined by the behaviour of matter on a cosmic scale. We may call this the generalized Mach's principle.

What we most desire is to find a quantitative relation which represents the generalized Mach's principle. To this end the crux of the matter is selecting a basic physical quantity that can characterize the fundamental behaviour of gravitationally bound systems on every scale. It is obvious that the 'action' is a preferred candidate, because the behaviour of known matter on a microscopic scale (a well-known typical scale in the Universe!) is characterized by Planck's action $\hbar$ (Planck's constant $\hbar$ divided by $2\pi$). Therefore, if the consideration given above is reasonable, then, in the simplest case, a general relation of the form

$$h^{(s)} = F(x, s)h^{(0)},$$

(3)

will be expected, where $s$ is the scale parameter, $h^{(s)}$ denotes the action which characterizes behaviour on the scale $s$, and $h^{(0)}$ denotes the position on the cosmic scale. The proportional factor $F(x, s)$ in relation (3) should be a dimensionless quantity which, in