A REMARK ON THE APSIDAL MOTION IN CLOSE BINARY SYSTEMS

(Letter to the Editor)

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(Received 29 April; revised 15 May, 1985)

Abstract. Dynamics of the apsidal motion in close binary systems are discussed. A comparison between the solution for the perfect fluid system and the solution for the rigid system reveals that some overall viscosity in the interior of distorted star has a right tendency to reconcile the observation of apsidal motion with the theory of internal structure.

1. Introduction

It has long been known that the observed value of the apsidal motion constant ($k_2$) is much smaller than the predicted value based on the theory of internal structure. This means that the rotation of the line of apside may be decelerated due to some unknown causes. In this connection, recently several authors tried to enhance concentration of density inside the stars in question without changing their observed locations on the H–R diagram. Their results are summarized by Sahade and Wood (1978). After all, for a number of systems the theoretical concentration was successfully enhanced by taking into account the revised opacity as well as the evolutionary effect. Nevertheless, the problem seems to be far from settled.

The stellar matter is supposed more or less viscous. For such a general case, as far as we are aware, the sole answer is given in Kopal's (1978) book. However, his treatment is limited to the systems with low viscosity and resulted in the same formula which Cowling (1938) and Sterne (1939) independently derived for the inviscid systems; and it is this formula which has been constantly utilized as a tool of the crucial test for the theory of stellar structure. In the following, a quick hint will be offered for investigating the effect of viscosity on the motion of a binary system.

2. Two Extreme Cases

Throughout this paper, the rotation axis of each component is assumed perpendicular to the orbital plane. In general, as long as the orbital eccentricity is small, the equation of elliptic motion can skillfully be solved as a special example of vibrations about the steady motion (Whittaker, 1937), which is here nothing but the circular motion, not only for the ordinary two-body problem but also for the close binary system affected by some perturbation. However, the subject is particularly concerned with internal friction or
some viscosity. One must then confront with tremendously complicated dynamics to solve. We shall, therefore, confine ourselves chiefly to two extreme systems, in the first the stellar matter being perfect fluid and in the second rigid. The perfect fluid component keeps the equilibrium figure instantaneously corresponding to the varying distance between two centers, whereas the rigid ellipsoidal component may give rise to some libration of each star in addition to the slight perturbation of orbital elements.

Fortunately, the answers for both cases are already known, the former was settled by Cowling and Sterne as mentioned above and the latter by Walter (1933). Simply it may be expected that the real answer could be found out between the two. Cowling and Sterne's formula can be written as

$$
\frac{\dot{\omega}}{n} = k_2 \left\{ \left( 1 + \frac{m_1}{m_2} \right) + 15 \frac{m_1}{m_2} \right\} \left( \frac{R_2}{a} \right)^5
$$

for the rate of precession of the apsidal line, where for the sake of simplicity, it was moreover assumed that the more massive component (designated 1 hereafter) be a sphere and the less massive one (hereafter designated by an index 2) an ellipsoid. Such a system could approximately be realized if component 1 is much more massive and smaller in size as in the case in the Algol system. The term inside the braces on the right-hand side of Equation (1) is separated into two parts, $1 + m_1/m_2$ from rotational distortion and $15 m_1/m_2$ from tidal distortion. Symbols $n$ and $\dot{\omega}$ denote the mean motion of relative orbit and the angular velocity of apsidal line, respectively, and $a$: the mean distance between both components, $m_1, m_2$ and $R_1, R_2$: masses and radii of components, respectively, $k_2$: the apsidal motion constant of component 2. Although binary components are usually both distorted, the effect of each component is simply additive as was shown by the previous authors.

On the other hand, under the rigid body assumption, Walter proved that the apsidal line should recede, and concluded his excellent work by the formula

$$
\frac{\dot{\omega}}{n} = \frac{3}{10} \frac{\mu_2}{a^2} \left( 3 \varepsilon_2^2 - \varepsilon_2^2 \right) \left( \frac{R_2}{a} \right)^2
$$

for the retardation rate, where $\varepsilon_2$ and $\varepsilon_2$ stand for the eccentricities of the equatorial section and of the principal meridional section of component 2, respectively. Symbol $\mu_2$ represents the ratio of $\Theta$ to $\Theta_0$, where $\Theta_0$ is the principal moment of inertia of a hypothetical homogeneous star which has the same mass, the same volume, and the same shape as the real component 2 and $\Theta$ the principal moment of inertia of ellipsoidal component 2. $\mu_2$ is another important indicator of the density concentration which is related to the apsidal motion constant $k_2$ by the equation (e.g., Walter, 1933)

$$
\mu_2 = \frac{5}{3} \frac{2 k_2}{2 k_2 + 1}.
$$

As $k_2$ varies from 0 for a Roche model to $\frac{3}{4}$ for a homogeneous star, $\mu_2$ changes from 0 to 1.