MATERIAL COEFFICIENTS AND TRANSFER OF
POLARIZED RADIO RADIATION IN A PLASMA

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Abstract. By a perturbation and diagram resummation method, a transport equation for the transverse field polarization matrix is established. This equation is then transformed into an equation for the Stokes parameters of the radiation. The equation takes the usual form of a transfer equation: the absorption and emission coefficients are matrix, the elements of which are given as a function of the dissipative part of the microcurrent correlation tensor and conductivity tensor. Finally this equation is expressed as a system for the intensities of the proper modes. The equations of the system are usually coupled.

1. Introduction

This paper is a continuation of a previous work (Heyvaerts, 1967) in which an approximate expression of the emission and absorption coefficients of a non-equilibrium plasma was given using a perturbation theory of the Liouville equation. Collective effects in this case were neglected. A first attempt to include collective effects has been made by Bel and Mangeney (1967). The basic idea consisted in the resummation of a whole class of irreductible diagrams; the dispersion relation of the medium then appeared as a result of the summation of a geometrical series. However the authors failed to consider all the relevant diagrams, so that we present here an alternative method of summation. In this case, however, the dispersion relation no longer appears in the results. But a simple mathematical trick allows us to switch to a form in which collective plasma modes are clearly apparent. In Section 2, the choice of the relevant diagrams and the method for resummation is outlined. In Section 3 expressions are derived for the microcurrent correlation tensor and conductivity tensor, which are used to establish the transport equation for the Stokes parameters in Section 4. In Section 5 this equation is converted into an equation for the proper modes and an example is given of a case in which this system of equations decouples.

2. Resummation Method

In the formalism we use, the plasma and the radiation field are described by a common phase density function. Cyclic boundary conditions are used, and the so called thermodynamic limit is taken, in which the total number of particles $N$ and the volume $V$ of the box in which the system is contained tend simultaneously to infinity, subject to the condition $n = N/V = C_{\text{re}}$. The transverse microscopic electromagnetic field is described by a discrete set of oscillators (Heitler, 1954) which are labeled by an index $\lambda$. This index represents both the wave vector $K$ of the oscillator and polarization $\mu$.
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The polarization vector is denoted by \( \mathbf{e}_\lambda \). Occasionally we shall replace \( \lambda \) by the more explicit index \((\mathbf{K}, \mu)\). Two dynamic variables are attached to each oscillator, \( \xi_\lambda \) and \( \eta_\lambda \), which are, the phase and the amplitude of the microscopic electromagnetic field. These dynamic variables evolve according to equations which can be put in an Hamiltonian form: they are equivalent to the Maxwell equations.

The particles are described by their positions and momenta, \( \mathbf{r}_j, \mathbf{p}_j \), when no magnetic field is present, and by the guiding centre position \( \mathbf{R}_j \), phase of gyration, \( \theta_j \), Larmor radius \( a_j \), and parallel momentum \( p_{j\parallel} \) when some uniform external magnetic field is present. The statistics of the oscillator's and particles dynamic variables is described by a density matrix \( D(\mathbf{R}_j, \theta_j, a_j, p_{j\parallel}, \xi_\lambda, \eta_\lambda) \) which obeys an evolution equation derived from the Liouville equation for the corresponding canonical variables. In this formalism the transverse electric and magnetic fields are written as follows:

\[
\mathbf{e} = \sum_\lambda 2\gamma_\lambda^{1/2} V^{-1/2} \mathbf{e}_\lambda \eta_\lambda^{1/2} \sin(2\pi \xi_\lambda - \mathbf{K}_\lambda \cdot \mathbf{r}),
\]

\[
\mathbf{b} = \sum_\lambda 2\gamma_\lambda^{1/2} V^{-1/2} \frac{\mathbf{K}_\lambda \times \mathbf{e}_\lambda}{|\mathbf{K}_\lambda|} \eta_\lambda^{1/2} \sin(2\pi \xi_\lambda - \mathbf{K}_\lambda \cdot \mathbf{r}),
\]

where, \( \mathbf{e}_\lambda \) is the unitary polarization vector associated with the oscillator \( \lambda \), and \( \gamma_\lambda = c|\mathbf{K}_\lambda| = c\gamma_\lambda \).

Now, we can define the tensor

\[
\pi_{ij} = \frac{1}{8\pi} \iint_V d^3r (e_ie_j + b_ib_j),
\]

whose trace is the total transverse electromagnetic energy

\[
Tr\pi_{ij} = \frac{1}{8\pi} \int_V d^3r (e^2 + b^2).
\]

By use of (1), the tensor \( \pi_{ij} \) can be brought to the form

\[
\pi_{ij} = \frac{c|\mathbf{K}|}{2\pi} \eta_{\mathbf{K}_1, \mathbf{K}_2} \exp[-2i\pi(\xi_{\mathbf{K}_1} - \xi_{\mathbf{K}_2})] \eta_{\mathbf{K}_1, \mathbf{K}_2} \exp[+2i\pi(\xi_{\mathbf{K}_1} - \xi_{\mathbf{K}_2})] \]

\[
\exp[-2i\pi(\xi_{\mathbf{K}_1} - \xi_{\mathbf{K}_2})] \eta_{\mathbf{K}_1, \mathbf{K}_2} \exp[+2i\pi(\xi_{\mathbf{K}_1} - \xi_{\mathbf{K}_2})] \]

Re designates the real (or hermitian) part, of a complex number (or tensor). Finally we define the tensor

\[
u_{ij}(\mathbf{K}) = \frac{\gamma_\lambda^3}{16\pi^3 c^3} \eta_{\mathbf{K}_1, \mathbf{K}_2}^{1/2} \exp[-2i\pi(\xi_{\mathbf{K}_1} - \xi_{\mathbf{K}_2})] \eta_{\mathbf{K}_1, \mathbf{K}_2}^{1/2} \exp[+2i\pi(\xi_{\mathbf{K}_1} - \xi_{\mathbf{K}_2})] \]

\[
\exp[-2i\pi(\xi_{\mathbf{K}_1} - \xi_{\mathbf{K}_2})] \eta_{\mathbf{K}_1, \mathbf{K}_2}^{1/2} \exp[+2i\pi(\xi_{\mathbf{K}_1} - \xi_{\mathbf{K}_2})] \]

\[
(3)
\]