NONLINEAR PROPAGATION OF WHISTLERS IN THE
IONOSPHERE

B. CHAKRABORTY, G. C. DAS, A. K. SUR, and S. N. PAUL*
Department of Mathematics, Jadavpur University, Calcutta, India

(Received 1 August, in revised form 27 December, 1985)

Abstract. In this paper, the nonlinear dispersion relation for whistlers in the ionosphere has been derived and then the group travel time for an ion-cyclotron whistler from its source to an observer at the satellite has been theoretically calculated. It is seen that the nonlinear effect has some important contribution in the expression of group travel time. Our present analysis gives a more correct result than that obtained by Gurnett and others. From numerical estimations, it is found that the group travel time of whistler may be changed reasonably due to nonlinear interaction of the wave and the plasma of ionosphere.

1. Introduction

Theoretical and experimental investigations on whistlers have been found to be interesting as well as important to understand the physical nature of phenomena occurring in the ionosphere. Gurnett et al. (1965) were the first to develop the basic theory of ioncyclotron whistlers in the ionosphere. Following their works, Gurnett and Brice (1966), Gurnett and Shawhan (1966), Singh and Tolpadi (1975), Das et al. (1984b) discussed rigorously the uses of proton whistlers as a diagnostic technique which is based on the measurement of group travel time for the whistler from its source to an observer at the satellite. Das et al. (1984a) pointed out that the measurement of group travel time may also give some new information about other astrophysical phenomena. All of these investigators are found to have discussed only the propagation of whistlers under the linearized theory. But, nonlinear interaction of waves in plasma (Sturrock, 1957; Jackson, 1960; Das, 1971; Chakraborty et al., 1984; and others) and so also in ionosphere exhibits many important and fascinating characteristics of dispersive waves (Matsumoto and Kimura, 1971; Karpman, 1974a, b; Brinca, 1981; Das, 1983). In the present paper, our motivation is to develop the mathematical theory of whistlers considering nonlinear interaction of waves with the medium. We have derived here the nonlinear dispersion relation of a circularly polarized wave propagating through ionospheric plasma and then calculated the group travel time of a proton whistler. From a numerical and graphical analysis we have shown the variation of group travel time with the nonlinearity in the medium. Finally, the results have been compared with these obtained by Gurnett and Shawhan (1966) and Gurnett and Brice (1966). The consequences of the nonlinear interaction of a wave with the medium are found to be significant in the propagation of whistlers and also on the group travel time of it. The group travel time may be increased by 6–33% due to nonlinear interaction of whistler

* Permanent address: Serampore Girls’ College, Serampore, Hooghly, West Bengal, India.

© 1986 by D. Reidel Publishing Company
with the plasma in the ionosphere, under some physical conditions. The parameter, measured by diagnostic techniques using whistlers, would now have different values which are more accurate than those obtained by previous workers (Das et al., 1985).

2. Basic Equations and Derivation of the Dispersion Relation

We assume that the ionospheric plasma, consisting of single species of ion (subscript \( i \)) having mass \( m_i \), velocity \( v_i \), number density \( n_i \), together with the electron (subscript \( e \)) with \( m_e, v_e, \) and \( n_e \), denoting the mass, velocity, and number density, respectively, is cold, homogeneous, and collisionless. The basic equations governing the plasma dynamics are:

\[
\frac{\partial v_x}{\partial t} + (v_x \cdot \nabla)v_x = \frac{q_x}{m_x} \left[ \frac{E + v_x \times H}{c} \right],
\]

\[
\frac{\partial n_x}{\partial t} + \nabla \cdot (n_x v_x) = 0,
\]

\[
\nabla \times E = -\frac{1}{c} \frac{\partial H}{\partial t},
\]

\[
\nabla \times H = -\frac{1}{c} \frac{\partial E}{\partial t} + \frac{4\pi}{c} \Sigma n_x q_x v_x,
\]

\[
\nabla \cdot E = 4\pi \Sigma n_x q_x,
\]

\[
\nabla \times H = 0;
\]

where \( x = i, e \); \( q_x = e \), when \( x = i \) and \( q_x = -e \), when \( x = e \). All the other symbols have their usual meanings.

To derive the dispersion relation, we assume that the variable parameters are perturbed as

\[
E = \varepsilon E^{(1)} + \varepsilon^2 E^{(2)} + \varepsilon^3 E^{(3)} + \cdots,
\]

\[
H = H^{(0)} + \varepsilon H^{(1)} + \varepsilon^2 H^{(2)} + \varepsilon^3 H^{(3)} + \cdots,
\]

\[
v_x = \varepsilon v_x^{(1)} + \varepsilon^2 v_x^{(2)} + \varepsilon^3 v_x^{(3)} + \cdots,
\]

\[
n_x = n^{(0)} + \varepsilon n_x^{(1)} + \varepsilon^2 n_x^{(2)} + \varepsilon^3 n_x^{(3)} + \cdots,
\]

where the parameters with superscript '0' represent the equilibrium values and (1), (2), (3) represent first-order, second-order, third-order, etc., perturbed values of respective quantities. \( \varepsilon \) is an arbitrary expansion parameter retained mainly for our convenience.

We further assume that the whistler is purely transverse and circularly polarized. Moreover, the first-order whistler has the form

\[
E_x^{(1)} = a(e^{i\phi} + e^{-i\phi}),
\]