SELF-MODULATION OF PULSAR RADIATION IN STRONGLY MAGNETIZED ELECTRON-POSITRON PLASMAS

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(Received 1 March, 1985)

Abstract. A nonlinear Schrödinger equation is obtained for linearly polarized electromagnetic waves propagating across the ambient magnetic field in an electron-positron plasma. The nonlinearities arising from wave intensity induced particle mass modulation, as well as harmonic generation are incorporated. Modulational instability and localization of pulsar radiation are investigated.

Present models (Sturrock, 1971; Ruderman and Sutherland, 1975) suggest that a strongly magnetized electron-positron plasma is formed near the polar caps of the neutron star. The investigation of coherent radio emission from the pulsar magnetosphere is receiving considerable interest (Cordes, 1979; Michel, 1982; Lominadze and Pataraya, 1982; Lominadze et al., 1982, 1983). Recent works (Chian and Kennel, 1983; Mofiz et al., 1984) presented nonlinear calculations for the pulsar microstructure. Specifically, Chian and Kennel (1983) suggested that the modulational instability of linearly as well as circularly polarized electromagnetic waves can give rise to localized pulses in an unmagnetized electron-positron plasma. Since the previous investigations (Chian and Kennel, 1983; Mofiz et al., 1984) did not account for the ambient magnetic field effects, application of the unmagnetized soliton theory to the pulsar radiation is limited. Recently, Yu et al. (1984) and Yu and Rao (1985) investigated the nonlinear propagation of circularly polarized pulsar radiation along a very strong ambient magnetic field. Large amplitude field aligned solitary envelope pulses were shown to exist.

On the other hand, most of the data (Ruderman and Sutherland, 1975; Cordes, 1979) show that the observed pulsar radiation is usually coherent and linearly polarized. In this paper, we investigate the modulational instability of the pulsar radiation whose electric field is polarized along the ambient magnetic field in an electron-positron plasma.

Consider the nonlinear propagation of linearly polarized pulsar radiation in the form

\[ E = zE \cos \varphi, \]  

where \( \varphi = \omega t - kx \), and the ambient magnetic field \( B_0 \) is directed along the z-axis. In the linear limit, the density perturbation associated with the ordinary mode is zero because the radiation electric field \( E \) is aligned along \( B_0 \). In the electron-positron plasma, the frequency \( \omega \) and the wavenumber \( k \) are related by

\[ \omega^2 = 2\omega_p^2 + k^2c^2, \]

where \( \omega_p \) is the plasma frequency of the electron-positron plasma.
where $\omega_p = (4\pi n_0 e^2/m_0)^{1/2}$ is the electron plasma frequency; $n_0$, the average plasma density; $m_0$, the rest mass of the electron; and $c$, the velocity of light.

Combining the Maxwell equations

\[ \nabla \times \mathbf{E} = -c^{-1} \partial_t \mathbf{B}, \]  
\[ \nabla \times \mathbf{B} = (4\pi/e)j + c^{-1} \partial_t \mathbf{E}, \]  

one can readily derive the wave equation

\[ (\partial_t^2 - c^2 \partial_x^2)E \cos \phi = -4\pi \partial_i j_x, \]

where $j_x = ne(v_{px} - v_{ex})$ is the field aligned plasma current, and $n_e = n_p = n$ is the particle number density.

The lowest order radiation induced velocity perturbations are determined from the parallel component of the momentum balance equation. One finds for species $j$ ($j = e$ for the electron, $j = p$ for the positron)

\[ v_{jz}^{(1)} = \frac{q_j E}{m_0 \omega} \sin \phi, \]

where $q_e = -e$ and $q_i = e$ are the electron and positron charge, respectively.

The change of the quiver velocity appears due to the relativistic particle mass variation. Under the weak relativistic approximation ($v_{jz}^{(1)} \ll c$), nonlinear particle velocity is given by

\[ v_{jz}^{(2)} \approx -\frac{c}{2} \left( \frac{q_j E \sin \phi}{m_0 \omega c} \right)^3. \]

The particles oscillating in the pulsar field would generate high-frequency currents which would then beat with the radiation magnetic field $\mathbf{B} = B_x \hat{y}$. The resulting nonlinear Lorentz force ($v_{jz}^{(1)} B_y$) then drives the plasma particles in a plane perpendicular to $\mathbf{B}_0$. Subsequently, nonlinear particle density fluctuations are generated which are determined by

\[ \partial_t \partial_n_j + n_0 \partial_x v_{jx} = 0, \]
\[ \partial_t v_{jx} = \Omega_j v_{jy} - \frac{q_j}{m_0 c} v_{jz}^{(1)} B_y, \]
\[ \partial_t v_{jy} = -\Omega_j v_{jx}, \]

where $\Omega_j = q_j B_0/m_0 c$ is the gyrofrequency of particle species $j$ and $B_y = -(ck/\omega)E \cos \phi$. Note that an electron-positron plasma does not support charge separation which may give rise to an ambipolar electric field. Combining Equations (9) and (10), one gets

\[ (\partial_t^2 + \Omega_j^2) v_{jx} = -\frac{q_j}{m_0 c} \partial_t (v_{jz}^{(1)} B_y). \]