A MODEL OF FLARE-PRODUCED MAGNETO-RADIATIVE
SHOCK WITH INCREASING ENERGY

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Abstract. An exact solution for a spherically-symmetric model of a magneto-radiative shock wave in the solar wind caused by the explosive energy release of a solar flare has been obtained in the case when energy released is an increasing function of the time. It has been shown that due to increasing energy, density, pressure, radiation flux, magnetic field and shock velocity change considerably.

1. Introduction

The problem of violent spherical explosion in a uniform atmosphere has been solved numerically by Taylor (1950) and Sedov (1945), by use of similarity variables under the assumption that the total energy of the flow between the shock and the point of explosion is constant. This assumption does not hold good for the flows driven by moving piston, since the work done by the pressure forces on the piston surface contributes to increase the energy of the flow between shock and the piston. Similarity flows for energy increasing with the time were studied by Jones (1955), Lee and Kubota (1957), Rogers (1958), Parker (1961) and Ranga Rao and Chaturani (1970).

In order to study the astrophysical problem of flare produced shock in the solar wind, Parker (1963) used the similarity transformation of Courant and Friedrichs (1948) to construct a model of hydrodynamic blast wave. This model corresponds to the flow due to a spherical piston expanding so that its radius varies as a power of the time. Lee and Chen (1968) attempted the only reasonably self-consistent similarity-variable model of a flare generated magneto-hydrodynamic shock. However, in this model the choice of similarity variable and the approximations made to produce a similarity scheme necessitated that the shock be of a rather special type with the velocity of propagation constant and wave energy increasing linearly with time.

Summers (1975) has studied magnetohydrodynamic blast waves in the solar wind caused by the explosive energy release of a solar flare under the assumption that the energy released is constant. Radiation effects and increasing energy have been taken into account in Summers' model. Since -- in the case of intense, prolonged solar flare activity -- the wave is driven by fresh erupting solar plasma for some time, its energy tends to increase as it propagates. In order to render the problem tractable we take a number of simplifying assumptions. The shock resulting from the flare eruption is assumed to result from a point explosion in the space and time accompanied by energy release $E$. It is also assumed that the
gas moves radially away from the point of explosion. We also take the inter-
stellar medium to be motionless, since otherwise it would cause destruction of
the shock's spherical symmetry and commencement of radial flow. The radiation
pressure and radiation energy are considered to be small in comparison to gas
pressure and energy respectively and therefore only radiation flux is taken into
account. The product solutions of McVittie (1953) have been used to evaluate
velocity, density, pressure, magnetic field and radiation flux.

Ahead of the shock the density distribution \( \rho_0 \) is taken to vary as an inverse
power of radial distance from the point of explosion – i.e.,

\[
\rho_0 = \frac{\rho_c}{r^{s}}, \quad (0 \leq s \leq 2) \quad (1.1)
\]

where \( \rho_c \) and \( s \) are constants.

In order to give a meaning to the otherwise physically unrealizable magnetic
field with spherical symmetry, the magnetic field is replaced by an idealised field
such that the lines of force lie on a hemisphere whose centre is the point of
explosion. The field is taken (cf. Rosenau and Frankenthal, 1976) as

\[
h_0 = \frac{h_c}{r}, \quad (1.2)
\]
directed tangentially to the advancing shock front, where \( h_c \) is constant.

The explosive energy released in solar flare on the surface of the Sun is
supposed to be time dependent and is given by

\[
E = E_c t^q, \quad (q > 0) \quad (1.3)
\]

where \( E_c \) and \( q \) are constants.

2. Equations of Motion

In the following equations expressed, in spherically-symmetric Eulerian form,
the viscosity and thermal conductivity are omitted and it is assumed that the
fluid has an infinite electrical conductivity (cf. Summers, 1975; or Rosenau and
Frenkenthal, 1976)

\[
\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \frac{2 \rho u}{r} = 0, \quad (2.1)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{h}{\rho r} \frac{\partial}{\partial r} (hr) = 0, \quad (2.2)
\]

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + h \frac{\partial u}{\partial r} + \frac{hu}{r} = 0, \quad (2.3)
\]

\[
\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left( \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} \right) + \frac{1}{\rho r^2} \frac{\partial}{\partial r} (Fr^2) = 0, \quad (2.4)
\]

where \( \rho, u, p, h \) are density, radial velocity, pressure and magnetic field behind