THE ELLIPSOIDAL VARIABLES

II. The Main-Sequence Distribution

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Abstract. We perform a Monte Carlo analysis of the eclipse probabilities of short-period binary systems in an attempt to estimate the distribution of ellipsoidal variable systems with Main-Sequence components. Our results suggest that possibly as many as one in four of the non-eclipsing, spectroscopic binaries with \( P(d) < 10 \), and Main-Sequence components with \( M_v < 10 \) could be ellipsoidal variable systems. This result, while an upper limit, would seem to hold irrespective of primary spectral type (B5 to G5).

1. Introduction

In Beech (1985) – hereafter referred to as Paper I – we considered the consequences of close stellar proximity. For binaries of period \( \leq 5 \) days it was suggested that the proximity effects would begin to become appreciable and that even if the geometry was such that no eclipses occurred the systems might still be expected to show ellipsoidal variability as a result of the variable stellar cross-section presented to the observer.

The ellipsoidal variables form a small subset of the short-period non-eclipsing, spectroscopic binaries. At present some twenty-seven such systems have been identified and of these fourteen are observed to have Main-Sequence components (Paper I), and it is these that concern us here. Since Main-Sequence systems are not highly evolved their phase space of analysis \( P^{\text{MS}} \) can be spanned by the quantities, mass ratio \( q \), period \( p \), semi-major axis \( A \), eccentricity \( e \), and inclination \( i \). The phase space of the ellipsoidal variables with Main-Sequence components, \( P^{\text{MS}}_{\text{ELL}} \), will be such that \( P^{\text{MS}}_{\text{ELL}} \subset P^{\text{MS}} \), and it is upon this understanding that we proceed in our analysis.

2. Procedure

In order to investigate the Main-Sequence distribution of the ellipsoidal variables we perform a Monte Carlo analysis of the eclipse probabilities of short period binary systems, based on the empirical Main Sequence of Budding (1981).

In our analysis we take a 50% binary distribution irrespective of spectral type and following Abt (1983) infer that 10% of all binaries have periods \( 1 \leq P(d) \leq 10 \). The relative distribution, \( \phi(sp) \), of the spectral types in our sample is calculated according to

\[
\phi(sp) \propto N^3(sp)m^{-\gamma(sp)} \left( \frac{dm}{dsp} \right),
\]

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where \( N(sp) \) is the number of stars of spectral type \( sp \) which have a magnitude less than \( M_\nu \), calculated according to

\[
5 \log \{N(sp)\} = 5 - 1.25A_\nu + M_\nu(sp) - M_\nu,
\]

(2)

\( m(sp) \) is the mass corresponding to spectral type \( sp \) and \( dm/dsp \) is the mass variation within a given spectral type calculated according to Table I of Budding (1981). The exponent \( y \) in (1) is the stellar mass distribution index, taken to be 2.35. The galactic extinction \( A_\nu \) was taken as 1 mag kpc\(^{-1}\).

\[
\begin{array}{lccccc}
\text{Sp. type} & N & \% \text{Binaries} & P(d) > 10 & P(d) < 10 & \text{Ellipsoidals} \\
B5 & 280 & 44.9 & 4.6 & 1.6 \\
A0 & 330 & 45.3 & 5.5 & 1.6 \\
A5 & 180 & 44.2 & 6.7 & 3.0 \\
F0 & 100 & 43.6 & 4.4 & 1.4 \\
F5 & 50 & 45.2 & 2.8 & 0.4 \\
G0 & 30 & 42.0 & 5.3 & 2.0 \\
G5 & 10 & 46.0 & 2.0 & 0.0 \\
\end{array}
\]

Table I

The model distribution of binary systems with primaries from spectral type B5 to G5. \( N \) is the number of stars from which the model systems are formed.

From Equation (1) we can determine the distribution of stars according to spectral type, and taking some 1000 stars between spectral types B5 to G5 we perform the experiment outlined in Figure 1. The aim of the procedure is to create binary systems and to estimate how many might be identifiable as ellipsoidal variables. All the binary systems generated have Keplerian \( (e = 0) \) orbits, which in the case of the observed ellipsoidal systems seems to be well satisfied, since for these, \( \langle e \rangle = 0.05 \pm 0.04 \). The mass ratios of the model systems are determined with respect to a bimodal distribution as determined by Trimble (1974) in the manner of Halbwachs (1981).

On selection of a \( P(d) < 10 \) binary a separation \( A(R_\odot) \) is selected at random from a constant distribution subject to the constraint that the period \( P(d) \) given by

\[
P(d) = 0.12(A/R_\odot)^{3/2}/\{(m_p/m_\odot)(1 + 1/q)\}^{1/2},
\]

(3)

is constrained by \( P_{\text{min}}(q) \leq P(d) \leq 10.0. P_{\text{min}}(q) \) corresponds to the separation being the sum of the component radii. Once a random inclination has been selected the radial velocity amplitude of the primary is determined according to

\[
K \text{ (km s}^{-1}\text{)} = 214q(1 + q)^{-2/3}(M_p/P(d))^{1/2}\sin i.
\]

(4)

The eclipse condition is calculated according to Equations (14) and (15) of Paper I,