DISTRIBUTION OF THE DIFFUSE RADIATION IN A HOMOGENEOUS ISOTROPIC ABSORBING SPECIMEN

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A method is given for calculating the spatial density of diffuse monochromatic radiation after penetration into a body of arbitrary shape. Results are presented for an infinite rod of rectangular cross-section.

Laser research often requires a knowledge of the distribution of the pumping radiation over the volume of the working substance, e.g., a cylinder, as in [1-3]. However, specimens of other shapes are often used, and the problem has not been solved for these. Here I derive a general expression for the intensity in a body of any shape; results are given for a rod of rectangular cross-section.

It is assumed that the incident radiation is perfectly diffuse, monochromatic, and incoherent, and also that the absorption coefficient is the same for all points and intensities. We may then use the laws of photometry, the intensity of the pumping radiation being taken as proportional to the spatial density of the energy, which is [4]

$$u' = \frac{1}{v} \int_{\Omega'} B'd\omega,$$

in which $v$ is the velocity of light in the material and $B'$ is the brightness of the field of view seen in the direction of the axis in an elementary solid angle $d\omega$.

Let $S$ be the closed smooth surface of the body (refractive index $n$, absorption coefficient $k$), which is exposed to diffuse light of intensity $B$. We convert (1) to a surface integral. We take an element $dS$ of $S$ at point $A$ and surround a point $P$ within the body with an elementary sphere (Fig. 1a). The equatorial section of the sphere perpendicular to the direction $L = AP$ is denoted by $d\omega$. The definition of $u'$ implies that the energy density at $P$ produced by the light entering via $dS$ is

$$du' = \frac{1}{v} \frac{dF}{d\omega},$$

in which

$$dF = \frac{B'}{L^2} \cos i'dS'd\omega,$$

the radiation flux in the tube formed by $dS$ and $d\omega$, $i'$ is the angle between the normal to $dS$ and $AP$, and $B'$ is the intensity of this entering light. But [5]

$$B' = n^2 (1 - r) B,$$

in which

$$r = \begin{cases} \frac{1}{2} \left[ \frac{\sin^2(i - i')}{\sin^2(i + i')} + \frac{\tan^2(i + i')}{\tan^2(i - i')} \right], & i' < i'_{cr}, \\ 1, & i' \geq i'_{cr} \end{cases}$$

is the Fresnel reflection coefficient, $i = \arcsin(n \sin i')$ is the angle of incidence, and $i'_{cr} = \arcsin \frac{1}{n}$ is the critical angle. The absorption in the path $L$ gives us with (2) and (3) that

$$du' = \frac{n^2}{\mu L^2} (1 - r) Be^{-kL} \cos i'dS.$$

Expression (5) still does not give the total energy, because not only the ray $AP$ from this element $dS$ passes to $P$.
but also light from other elements. Figure 1 shows that \( P \) receives also ray \( A_1A_P \) from \( A_1 \) reflected at \( A_2 \), ray \( A_2A_1A_P \) from \( A_2 \) reflected at \( A_1 \) and \( A_2 \), and so on. The paths of all these rays may be determined unambiguously by computing the reverse ray \( PAA_1A_2 \ldots \). The positions of \( A \) and \( P \) are given, so ray \( PA \) is as well; its further course is governed by the law of reflection and the shape of surface \( S \). The process involves calculation of the angles \( i', i'_{1}, i'_{2}, \ldots \) between the ray and the normals at \( A, A_{1}, A_{2}, \ldots \), as well as calculation of lengths \( D_{1}, D_{2}, \ldots \) between the points. These angles are used in (4) to find the reflection coefficients \( r = r(i'), i'_{1} = r(i'_{1}), i'_{2} = r(i'_{2}), \) while the lengths are used to determine \( L_{1} = L + D_{1}, L_{2} = L_{1} + D_{2}, \ldots \), in which \( L_{1} \) is the path of a ray undergoing \( m \) reflections.

Consider first the case of a single reflection. The light reflected from \( dS \) produces at \( P \) a radiation density
\[
\frac{dU'}{dV} = \frac{dF_{1}}{vdV},
\]
in which \( dF_{1} \) is the flux reaching \( P \) after refraction at \( A_{1} \) and reflection at \( A_{1} \); it is found by producing to \( A_{1} \) the tube \( dS \) formed by \( do \) and \( dS \), which gives us the element \( dS_{1} \) formed by the tube in \( S \) around \( A_{1} \). We take \( S \) as planar and construct the mirror image \( A_{1}A_{1} \) of ray \( A_{1}A_{1} \) formed by \( dS \). This gives us the straight tube \( PAA_{1}A_{2} \) (Fig. 1b).

\( dF_{1} \) equals the part of the flux entering through \( dS_{1} \) that does not escape from tube \( AP \) having \( dS \) and \( do \) as elements. Let \( d\omega_{1} \) be the solid angle formed by \( do \) at \( dS_{1} \):
\[
d\omega_{1} = d\sigma/L_{1}^{2}.
\]

Figure 1b shows that \( d\omega_{1} \) is constant only within the central part of \( dS_{1} \), whereas vignetting by \( dS \) causes it to fall to zero at the edge; it is not given by (7). This feature hinders deduction of \( dF_{1} \), but \( dS \) and \( d\sigma \) are independent, so we may legitimately suppose that \( d\sigma \ll dS \), in which case vignetting may be neglected, so \( dF_{1} \) is found as
\[
dF_{1} = B_{1} \cos i'_{1} dS_{1} d\omega_{1},
\]
in which
\[
B_{1} = n_{2} (1 - r_{1}) r_{B} e^{-kL_{1}},
\]
is the intensity of the beam as corrected for refraction at \( A_{1} \), reflection at \( A_{1} \), and absorption over a path \( L_{1} \). The \( dS_{1} \) of (8) may be taken as the area of intersection of \( S \) near \( A_{1} \) with a cone having the solid angle
\[
d\omega = dS \cos i'/L_{1}^{2},
\]
of \( dS \) seen at \( P \). Then
\[
dS_{1} = L_{1}^{2} \cos i'/L_{1}^{2} dS.
\]
Then (6)-(9) give
\[
\frac{dU'}{dv} = \frac{n_{2}^{2}}{vL_{1}^{2}} (1 - r_{1}) r_{B} e^{-kL_{1}} \cos i' dS.
\]

An analogous argument for a flux reflected \( m \) times gives
\[
dU_{m}' = \frac{n_{2}^{2}}{vL_{m}^{2}} (1 - r_{m}) \ldots r_{1} r_{B} e^{-kL_{m}} \cos i' dS,
\]

Formula (1) implies that \( B = \) constant outside the body and is
\[
u = \frac{4\pi B}{c},
\]
in which \( c \) is the velocity of light outside the body. The total energy density \( u' \) within the specimen is the integral over \( S \) of the sum of (6), (10), and (11). Completely diffuse illumination gives
\[
q = \frac{u'}{u} = \frac{n_{2}^{2}}{4\pi} \int \frac{\cos i'}{L_{1}^{2}} (1 - r) e^{-kL} + (1 - r_{1}) r_{B} e^{-kL_{1}} + \ldots +
\]

202