Approximate Analysis for Stationary Current Flow in Two-Dimensional Josephson Tunnel Junctions

R. Vaglio

Istituto di Fisica, Università di Salerno, Salerno, Italy

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The stationary current distribution in Josephson tunnel junctions is discussed within the framework of an approximate linear analysis. It is shown that such an approach can provide very useful information also on the behavior of two-dimensional junctions. Self-limiting effects and the magnetic field dependence of the critical current for various geometries are discussed in some detail. The special case of a square junction is analyzed extensively.

1. INTRODUCTION

It is well known that the behavior of a rectangular Josephson tunnel junction for the stationary case can be obtained in principle by solving, with the appropriate boundary conditions, the equation 1 (Fig. 1)

$$\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{1}{\lambda_J^2} \sin \varphi$$  \hspace{1cm} (1)

where $\varphi$ is the phase difference between the two superconductors and

$$\lambda_J = \left( \frac{\hbar}{2e \mu_0 J_1(\lambda_1 + \lambda_2 + t)} \right)^{1/2}$$

is the Josephson penetration depth, $\lambda_1$ and $\lambda_2$ are the London penetration depths, $t$ is the barrier thickness, and $J_1$ is the maximum current density.

The current density in the barrier is then given by

$$J(y, z) = J_1 \sin \varphi(y, z)$$  \hspace{1cm} (2)

and the field in the junction by

$$B(y, z) = \left( \frac{\hbar}{2ed} \right) \nabla \varphi(y, z)$$  \hspace{1cm} (3)

where $d = \lambda_1 + \lambda_2 + t$. 

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Unfortunately, Eq. (1) has been solved exactly only for a few simple geometries \(^2,^3\) ("in-line" and "asymmetric" one-dimensional junctions). In a recent paper \(^4\) an approximate method for solving Eq. (1) with rather general boundary conditions for one-dimensional junctions was proposed. This method provides simple approximate analytic expressions for the dependence of the maximum Josephson current \(I_J\) on the ratio \(L/A_1\) and on the external applied magnetic field \(B_\perp\).

In the present work the same kind of analysis as developed in Ref. 4 is generalized to include two-dimensional cases. The special case of a square junction is discussed extensively and the results are compared with experiments carried out by various authors.

2. THE LINEAR APPROXIMATION IN TWO DIMENSIONS

Let us assume, following Ref. 4, for the current–phase relation the approximate expression

\[
J(y, z) = \frac{1}{4} J_1 \varphi(y, z)
\]  

(this expression is sketched in Fig. 1c together with the correct sinusoidal relation).