CHAOTIC BEHAVIOR AND STATISTICAL ANALYSIS OF SOME MIRA AND SR STARS

T. YANAGITA
The Institute of Statistical Mathematics, 4-6-7 Minami-Azabu, Minato-ku, Tokyo 106, Japan

H. SATOH
Institute of Astronomy, The University of Tokyo, Mitaka, Tokyo 181, Japan

and

K. SAIJO
Department of Science and Engineering, National Science Museum, Ueno Park, Taitou-ku, Tokyo 110, Japan

Abstract. Visual light curves of some Mira and semiregular variable stars are analyzed to search for their chaotic and statistical behaviour. Discussion are also given.

1. Data

We analyze light curves of some Mira and semiregular (SR) variable stars, which are listed in table I, to find out chaotic and statistical behavior. Light curves of these stars are obtained from the database of Variable Star Observers League in Japan (Saijo and Kiyota 1991). Periods of long-term observations are about 20000 days for these stars except for W Cyg, whose observation period is about 4000 days.

Before analysis, we average the same day's observations and smooth out the fluctuation with high frequency by averaged over 20 day's ones. After these arrangement, we get the time series with its unit time, 20 days.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristics of Stars</td>
</tr>
<tr>
<td>Star</td>
</tr>
<tr>
<td>o Cet</td>
</tr>
<tr>
<td>χ Cyg</td>
</tr>
<tr>
<td>R Leo</td>
</tr>
<tr>
<td>V Boo</td>
</tr>
<tr>
<td>W Cyg</td>
</tr>
<tr>
<td>RS Cnc</td>
</tr>
</tbody>
</table>

From GCVS 4th edition (1985)
2. Statistical Analysis

To understand the statistical property of the time series, periodogram is a basic statistic and defined by Fourier transformation of time series. However, the periodogram analysis is difficult to determine the peak of spectrum. Therefore, we use Auto Regressive (AR) model to estimate the power spectrum (e.g. Akaike and Nakamura 1988). AR model expressed the value $x(t)$ as a linear combination of $M$ past values, $x(t) = \sum_{m=1}^{M} a(m)x(t - m) + \varepsilon(t)$ ($t=1,2,…$), here $\varepsilon(t)$ is Gaussian noise. The parameters, $a(m)$ and $< \varepsilon \varepsilon >$, are estimated by Yule-Walker method. After estimating the parameter of AR model, we determine the power spectrum by

$$ p(f) = \frac{\sigma^2}{|1 - \sum_{m=1}^{M} a(m) \exp(-i2\pi fm)|^2} $$

The main periods of the stars are easily determined because the power spectrum has a rational function form. The periods obtained from AR model are shown in table II.

### TABLE II

<table>
<thead>
<tr>
<th>Period Obtained by AR Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ Cet</td>
</tr>
</tbody>
</table>
| $\begin{array}{llll}
\text{Period} & \text{Power} \\
333.3 & 2.34 & 412.4 & 2.56 & 312.5 & 2.64 \\
171.7 & 1.61 & 205.1 & 2.17 & 168.8 & 1.35 \\
116.3 & 0.55 & 136.5 & 1.03 & 2105.3 & 1.21 \\
769.2 & & & & & \\
\end{array}$ |
| $\begin{array}{llll}
\text{Period} & \text{Power} \\
206.4 & 1.59 & 128.6 & -0.34 & 217.4 & -0.49 \\
135.6 & 0.25 & 264.9 & -0.38 & 130.7 & -0.56 \\
888.9 & 0.17 & & & 327.9 & -0.61 \\
\end{array}$ |
| $\begin{array}{llll}
\text{Period} & \text{Power} \\
256.4 & 1.59 & 128.6 & -0.34 & 217.4 & -0.49 \\
135.6 & 0.25 & 264.9 & -0.38 & 130.7 & -0.56 \\
888.9 & 0.17 & & & 327.9 & -0.61 \\
\end{array}$ |

3. Dynamical System Approach

The time series of Mira and SR stars behaves like a low dimensional chaotic dynamical system such as Lorenz system. Lorentz plot (return map of time interval between one local maximum and next local maximum) is useful to determine the low dimensional chaotic dynamics. However, in this case, this return map is difficult to plot because of missing observation. So we plot a stroboscopic return map, $x(t)$ vs $x(t + T)$. Stroboscopic return map of Mira stars has circular structure because of the periodic behavior, while that of SR stars shows more irregular map like a Brownian motion.