NONLINEAR ELECTROSTATIC DRIFT KELVIN-HELMHOLTZ INSTABILITY

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Abstract. Nonlinear analysis of electrostatic drift Kelvin–Helmholtz instability is performed. It is shown that the analysis leads to the propagation of the weakly nonlinear dispersive waves and the nonlinear behavior is governed by the nonlinear Burger’s equation.

1. Introduction

The Kelvin–Helmholtz instability in a plasma occurs when there is a relative motion between different layers. The fundamental of the instability lies in the fact that the pressure perturbation does work on the interface between the two layers. The instability investigation of the interface between two plasma layers of different velocities is important in several cosmical and laboratory situations, such as meteor entering the Earth’s atmosphere, when the air is blown over mercury, wind blowing over the ocean, in the theory of solar winds, studies of the Earth’s magnetosphere instability, the stream structure of solar corona, the helical wave motion observed in ionized comet tails, etc. Several researchers have investigated the Kelvin–Helmholtz instability in various configurations. Chandrasekhar, in his monograph, has given a good account of such investigations. Applications of drift Kelvin–Helmholtz instabilities have been discussed earlier by several authors such as Stix (1973) and Catto et al. (1973), who have studied the thermonuclear applications. The space applications of these instabilities have been discussed by Dobrowolny (1972), Dobrowolny and D’Angelo (1972), and Bavassano et al. (1978). The kinetic analysis of above instabilities in high-β plasma was performed by Mikhailovskii and Klimenko (1980). Two branches of magneto-acoustic instabilities and one branch of the Alfvén instability were discussed in the paper and results were compared with the hydrodynamic results given by D’Angelo (1965) and Dobrowolny (1972). In continuation, Mikhailovskii (1982) developed a hydrodynamic theory of drift Kelvin–Helmholtz instabilities including the gyro-radius effects and the hydrodynamic results were in complete agreement with kinetic ones. Mikhailovskii (1982) discussed in his paper the magneto-acoustic instabilities and the Alfvén instability. Nonlinear analysis of K–H instability of a rotating plasma was performed by Vyas et al. (1983). Furthermore, a weakly nonlinear theory of K–H instability for magnetic fluids was presented by Malik and Singh (1986). Sharma and Srivastava (1992a, b) have studied...
drift K–H magneto-acoustic instabilities and Alfvén instability in the presence of equilibrium electric field. One of the cases considered by Mikhailovskii (1982) was the electrostatic drift Kelvin–Helmholtz instability of a low-β plasma. We wish to perform the nonlinear analysis of drift Kelvin–Helmholtz instability for electrostatic perturbations.

The plan of the paper is as follows: the basic equations including the magnetic viscosity and ion collisionless heat flux derived by Mikhailovskii and Tsypin (1971) are given in Section 2. The nonlinear analysis is performed in Section 3. In Section 4 we discuss the nonlinear differential equation.

2. Formulation and Basic Equations

We consider a plasma strata consisting of two species, viz., electrons and ions and describe each species by density $n$, velocity $v$, and pressure $p$. The basic equations governing the problem are continuity equation

$$\frac{\partial n}{\partial t} + \text{div}(n\vec{v}) = 0,$$  

(1)

equation of motion

$$mn\frac{d\vec{v}}{dt} + \nabla \cdot \pi = -\nabla p + en \left( \vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right),$$  

(2)

and the heat balance equation

$$\frac{dp_i}{dt} - \gamma_0 T_i \frac{dn}{dt} + (\gamma_0 - 1) \text{div} \vec{q}_i + \gamma_0 \text{div} \vec{q}_\perp = 0,$$  

(3)

where $d/dt = \partial/\partial t + \vec{v} \cdot \nabla$ and $\vec{E}, \vec{B}$ are electric and magnetic fields, $e$ and $m$ are charge and mass of the particles and $c$ is the velocity of light. Our equations also contain $T = p/n$. In the discussion, ion and electron quantities are identified by $i$ and $e$, respectively; and quantities without indices refer to arbitrary particle species. In Equation (2) and Equation (3) $\pi$ and $q$ refer to viscosity tensor and heat flux, respectively, which are retained only in the case of ions, values of these quantities are given by Braginskii (1965).

Furthermore, we assume that the low-β plasma generates an irrotational electric field

$$\vec{E} = -\nabla \phi,$$  

(4)

$\phi$ being the scalar electrostatic potential. Also, the electron pressure is determined by the thermal conductivity equation

$$B \cdot \nabla T_e = 0$$

and

$$T_e = \frac{p_e}{n_e}.$$  

(5)