GENERALIZED ABELIAN HIGGS SUNSPOT ENDOWED WITH A ROTATORY MOTION

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Abstract. An Abelian Higgs model of sunspot generalized in a Chern-Simons-like fashion is discussed. It is shown, in particular, that both the magnetic and electric fields are present inside the sunspot, and that the latter rotates. One demonstrates that the total angular momentum of a static, cylindrically symmetric sunspot is proportional to $p^2$, where $p$ — an integer — stands for the number of magnetic flux quanta carried by the spot. Finally, the behaviour of the Higgs field amplitude, magnetic and electric field strengths are illustrated for the spots carrying one to five flux quanta, all having the penumbra-to-umbra radius ratio of the value $1.6\sqrt{2}$.

1. Introduction

Three years ago one of us (Saniga, 1990a, b) revealed the remarkable analogy between the sunspot and the type II superconductor magnetic vortex. This analogy was then subsequently exploited as a basis for developing the so called Abelian Higgs (henceforth referred to as AH) theory of sunspots (Saniga, 1990a, b; 1992a–d; 1993a, b; Saniga and Klačka, 1993a–c); the theory turned out to lead to a much deeper insight into the sunspots’ physics than it was possible to gain with the help of a currently used MHD framework. Thus our Abelian Higgs approach gave a natural explanation of the existence of both the spot’s umbra and penumbra; easily accounted for the confinement of the spot’s magnetic field into a finite-dimensional, sharply bounded flux tube as well as for its (topological) stability; implied the phenomenon of sunspots’ splitting; showed that the process of sunspot’s creation and/or dissolution may be regarded as a phase transition-like process; and - last but not least - led unambiguously to the filamentary structure of spots’ penumbras. The purpose of this contribution is to generalize the above theory in such way to provide us with the tool which would enable us to handle also spots endowed with a rotatory motion. As shown in detail below this might be achieved if we generalize the ordinary AH model in a Chern–Simons-like fashion (Saniga, 1993b).
2. Generalized Abelian Higgs Sunspot

Thus our starting point will be a Lagrangian density of the form

\[
\mathcal{I} = \sqrt{-g} \left[ -\frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} + \frac{1}{4} \Theta \varepsilon^{\mu\nu\rho} F_{\mu\nu} A_\rho + \frac{1}{2} \left( \frac{\partial \Phi}{\partial x^\rho} + igA_\rho \Phi \right) \times \right. \\
\times \left( \frac{\partial \Phi^*}{\partial x^\rho} - igA_\rho \Phi^* \right) g^{\rho\sigma} - \frac{\lambda}{4} \left( \Phi \Phi^* - \frac{m^2}{\lambda} \right)^2 \right],
\]

(1)

where the second term in square brackets is the Chern-Simons (CS) term (see, e.g. Jackiw et al., 1990). Here \( \Theta \) is a constant and for the explanation of the remaining symbols and for the notation the reader is referred to Saniga (1992b, d; 1993b); the last reference also contains a brief outline concerning a possible origin of the CS term in the context of solar physics.

It can easily be verified that the equations of motion following from (1) are

\[
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\kappa} \left( \sqrt{-g} F^{\rho\kappa} \right) = \frac{i}{2} \varepsilon^{\rho\mu\nu} \left( \Phi \frac{\partial \Phi^*}{\partial x^\kappa} - \Phi^* \frac{\partial \Phi}{\partial x^\kappa} \right) g^{\kappa\nu} + g^2 A^\rho A^\Phi \Phi^* + \\
+ \frac{\Theta}{2 \sqrt{-g}} F_{\mu \nu} \equiv - j^\rho + \frac{\Theta}{2 \sqrt{-g}} F_{\mu \nu},
\]

(2)

and, parameterizing \( \Phi(x) = f(x) \exp(i\varphi(x)) \),

\[
\frac{1/f}{\sqrt{-g}} \frac{\partial}{\partial x^\rho} \left( \sqrt{-g} g^{\rho\kappa} \frac{\partial f}{\partial x^\kappa} \right) = \\
= \left( \frac{\partial \varphi}{\partial x^\kappa} + ga_{\kappa} \right) \left( \frac{\partial \varphi}{\partial x^\rho} + ga_{\rho} \right) g^{\rho\kappa} - \lambda f^2 + m^2,
\]

(3)

\[
-1 \frac{\partial}{\sqrt{-g}} \frac{\partial}{\partial x^\rho} \left( \sqrt{-g} g^{\rho\kappa} \frac{\partial \varphi}{\partial x^\kappa} \right) = \\
= \frac{g}{\sqrt{-g}} \frac{\partial}{\partial x^\rho} \left( \sqrt{-g} g^{\rho\kappa} a_{\kappa} \right) + 2g^{\rho\kappa} \frac{\partial \ln f}{\partial x^\rho} \left( \frac{\partial \varphi}{\partial x^\kappa} + ga_{\kappa} \right),
\]

(4)

and show, like those describing the ordinary AH model, an invariance under gauge transformation. Since our intention is to keep things simple and, at the same time, as instructive as possible in what follows, we confine ourselves to the case of a flat (Minkowski) space-time and take the field configuration to be static (i.e. \( \varphi_0 = 0 \)). As the first step we examine geometric properties of such a configuration.