COSMIC BACKGROUND TEMPERATURE AND THE COUPLING CONSTANTS OF FUNDAMENTAL INTERACTIONS

(Letter to the Editor)

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Abstract. The underlying links between the coupling constants of the fundamental interactions and cosmological parameters explored earlier is shown to give rise to a definite value for the cosmic background temperature.

In a recent paper (Sivaram, 1982a; 1986), it was pointed out that the gross parameters characterising the universe, such as the overall size, can be arrived at from microphysical considerations involving the fundamental interactions of elementary particle physics. For instance, the Hubble radius of the universe was obtained in terms of the coupling constants of the four fundamental interactions:

\[ R_H = \frac{g^4}{G e^8} (c^7 G_F / \hbar)^{1/2} \simeq 10^{28} \text{ cm s}. \] (1)

Where \( G \) is the Newtonian gravitational constant, \( \hbar \) is the Planck's constant, \( c \) is the velocity of light and \( G_F \) is the universal Fermi weak interaction constant (= \( 1.5 \times 10^{-49} \text{ erg cm}^3 \)). The electro-magnetic and strong interactions are characterized by dimensionless coupling constants, \( e^2 / \hbar c = 1/137 \) and \( g^2 / \hbar c \simeq 14.5 \), the strong interaction pion-nucleon coupling. Several other interesting relationships connecting the parameters of cosmology and elementary particle physics were also given in other papers (Sivaram, 1982b; 1984). In particular, in Sivaram (1982b; 1986) a definite value for the total mass \( M \) of the universe was obtained as:

\[ M_U = \frac{e^4}{G^2 m_p m_e^2} \simeq 6 \times 10^{54} \text{ g} \] (2)

(where \( m_e \) is the electron mass). \( M_U \) can also be expressed in terms of the other coupling constants. In the paper (Sivaram, 1982b), the above \( M_U \) was arrived at from general relativistic considerations combined with electromagnetic laws of electron photon scattering while in the paper (Sivaram, 1986), the same value of \( M_U \) was found from requirements of consistency in the observed low energy values of the coupling constants of strong, electromagnetic and weak interactions in the

Machian type of unification of fundamental interactions and quantum physics with cosmology. Considering that the Hubble volume of the universe is given by

$$V_H = 2\pi^2 R_H^3,$$

(3)

Eqs. (1)–(3) would imply a baryonic density at the current epoch of

$$\rho_B \simeq \frac{e^4 G^3 \epsilon^{24}}{G^2 m_e^2 m_p} \frac{G^3 e^{24}}{2\pi^2 g^{12}} \left( \frac{\hbar}{c^7 G_F^3} \right)^{3/2},$$

(4)

This gives $$\rho_B \approx 3 \times 10^{-31} \text{ g cm}^{-3},$$ at the present epoch. If $$Y$$ denotes the fractional helium abundance, the present density of helium nuclei would be

$$\rho_{\text{He}} = Y \rho_B.$$

For $$Y \approx 0.25$$, this gives $$\rho_{\text{He}} \approx 7.5 \times 10^{-32} \text{ g cm}^{-3}$$. If $$\epsilon$$ be the energy released in the formation (i.e., nucleosynthesis due to thermonuclear actions) of one gram of helium ($$\epsilon = 6 \times 10^{18} \text{ erg g}^{-1}$$) and we make the assumption that the entire energy present in the cosmic microwave background is due to the total energy released in the reactions forming helium then the energy density in the microwave background at the present epoch is given by:

$$\rho_{\text{rad}} = \rho_{\text{He}} \epsilon = Y \rho_B \epsilon.$$

(5)

If $$T_B$$ be the temperature of this background radiation, then

$$\rho_{\text{rad}} = a T_B^4,$$

(6)

where $$a$$ the Stefan-Boltzmann radiation constant can be written

$$a = \frac{\pi^2 k_B^4}{15 \hbar^3 c^3},$$

(7)

where $$k_B = 1.38 \times 10^{-16} \text{ erg K}^{-1}$$ is the Boltzmann constant.

Now substituting for $$\rho_B$$ in Eq. (5), from the formula in Eq. (4), and then combining Eqs. (5), (6) and (7), finally gives the following expression for the cosmic background temperature.

$$T_B = \left( \frac{15 Y G \epsilon}{2 m_e^2 m_p} \right)^{1/4} \frac{1}{\pi K_B} \frac{e^7}{g^3} \left( \frac{\hbar^9}{c^{15} G_F^9} \right)^{1/8}.$$

(8)

This gives $$T_B = 2.8 \text{ K};$$ in good agreement with observations. A more precise estimate gives $$T_B = 2.74 \text{ K}$$ (Sivaram, 1992). The corresponding baryon to photon density is $$\eta \approx 3.9 \times 10^{-10}.$$