Abstract. The determination of the photon path-length distribution function (PLDF) for the case of linearly anisotropic (Rocard) scattering in a semi-infinite plane-parallel homogeneous atmosphere using the Piessens-Huysmans method is described in detail. It has been shown that in this case the PLDF may have a minimum at small path-lengths — a feature which is never encountered in isotropic scattering. The respective regions with minima in the ($\mu$, $\lambda$)-plane have been sorted out. As a rule, the average path-length in the case of the forward/backward Rocard scattering is larger/smaller than that in the isotropic case. The precise average path-lengths for a number of parameters are shown in Table I.

1. Introduction

In astronomy, the photon path statistics have been subject to numerous studies since the paper of Irvine (1964). Taking into account the close connection between the statistics and the time-dependent radiative transfer problems (Minin, 1988) according to Nagirner (1974), we may go back as far as the paper of Fock (1926).

The basic theoretical background for the photon path-length distribution function (PLDF) and the ways to determine it in a homogeneous semi-infinite plane-parallel atmosphere were discussed in the papers of Irvine (1964, 1965). Almost at the same time Romanova (1964, 1965) had obtained some results on the reflection and transmission functions for a layer of large but finite thickness by using an approximate formula of Rozenberg (1962) which described the reflection function as a function of the photon path-length $l$ for $l \gg 1$. The same asymptotic problem was considered by Ivanov and Gutshabash (1974) for the time-dependent transfer in a rigorous way.

One of the main causes of such an intensive interest is that when we are dealing with spectral lines and molecular bands where the albedo of single scattering $\lambda$ is a rapidly changing function of frequency, it would be useful to express the solution of the radiative transfer equation at different $\lambda$ values in terms of the solution for the conservative case. The photon path statistics approach gives us such a possibility, and this has been used to find the equivalent widths of different spectral lines and the respective curves of growth in planetary atmospheres (Fouquart and Lenoble, 1973; Fouquart, 1974).

Another way of using the photon statistics is to determine the mean number of scatterings in an atmosphere. This line of approach has been extensively studied.
by Ambarzumyan (1948) and Sobolev (1966a, 1966b, 1967). According to these studies we may simply estimate, e.g. the time a photon is spending to emerge from a gaseous medium (Ambarzumyan, 1948). Uesugi and Irvine (1970) have made use of the fact that in a semi-infinite medium the average path length and the mean number of scatterings are equal and they have computed the latter for a sample of albedos of single scattering and the angular variables $\mu$ and $\mu_0$.

There are, however, not many attempts to determine the photon path-length distribution function in the astronomical literature. Perhaps one of the best discussions of the subject is given in a paper by Appleby and Irvine (1973). They have considered a semi-infinite homogeneous atmosphere where the scattering was isotropic or governed by the phase functions approximating a maritime haze and a cumulus cloud. They have used the weighted summation of the intensities in different orders of scattering which were described by Uesugi et al. (1971) and the results were compared with those obtained by a Monte Carlo computation. They have given many samples of the PLDF while the parameters of the distributions varied in a broad range. One of their main results was the discovery of the fact that the shape of the PLDF depended markedly on the degree of forward scattering due to the phase function. The PLDF, both for haze and cloud phase functions, show quite marked minima at small path-lengths, although $p(\lambda, l)$ is not zero, at least for the set of parameters considered. The elongated nature of the phase function may be the reason for this, since we have encountered the presence of minima for the Rocard scattering, too.

We have already listed one of the possible ways to determine the PLDF — using the weighted summation of the intensities in different orders of scattering. Another approach has been used by Fouquart (1974) who exploited a method based on the Padé approximation.

Van de Hulst (1980) has shown an asymptotic expansion for large path-lengths which gives exceptionally good results in the region where other methods quite often fail.

Our aim is to study in detail how the linearly anisotropic (Rocard) phase function influences the PLDF and its first two moments in the case of a semi-infinite plane-parallel homogeneous atmosphere. An important result obtained is that in the case of Rocard scattering, the PLDF shows minima at small path-lengths — a feature which is never encountered in the case of isotropic scattering. In the $(\mu, \lambda)$-plane we have found the boundary curves of the regions where the minima of the PLDF are present. It appeared that for the case with $x = 1$ and $\phi = \pi$ the situation has strong resemblance to a specular reflection — at $\mu = \mu_0$ the radiation has no chance to reach the deep layers and there are no minima of the PLDF at small path-lengths.

We have confirmed the asymptotic behaviour of the average path-length at albedos of single scattering close to conservative as pointed out by Uesugi and Irvine (1970) and van de Hulst (1980), and found a similar asymptotic expression for the dispersion.