Abstract. In this paper, we make use of the Stumpff's functions to solve the problem of determining the orbit of a visual binary star in universal variables. The method is thus valid for all types of orbits: hyperbolic, parabolic and elliptic.

1. Introduction

The so-called Thiele-Innes-van der Boss (see e.g. [1]) was the first method for computing the elliptic orbit of a visual double star with the necessary and sufficient data, namely, the method makes use of three complete observations \((\rho, \theta, t)\) and of the double areal constant of the apparent orbit. Several modifications of this method have been made [9, 10]. Cid [2], based on the previous method, developed a new one considering only observational data: three complete observations \((\rho, \theta, t)\) and one incomplete one \((\theta_1, t_1)\). Later on, Docobo [8] obtained a new method more feasible, by analyzing the set of Keplerian orbits passing through three points.

Although most of the known orbits correspond to the elliptic case, in theory, there is no restriction to the fact that the orbit may be parabolic or hyperbolic. This is what motivated our work: to produce a general formulation for computing visual double stars orbits independently on the type of orbit considered.

Regularizing techniques are well-known in celestial mechanics since the last century and several sets of variables have been defined ever since and have been widely applied in Astrodynamics, Celestial mechanics, etc. For more information on them, the reader is referred to the extensive bibliography in [7].

In our work we make use of the so-called Stumpff functions [11, 12]. Following Correas [4, 5] notation, we proceed to the obtaining of an equation equivalent to the classical Thiele equation in universal variables. Once this equation (the basis of the Thiele-Innes-van der Boss method and its variants), we reformulate in universal variables the Cid's method [2] for it contains only observational data. In this way, we obtain the fundamental system (23), which solution will allow the computing of the orbital elements.

As an illustration, we present here the application of our method to the binary ADS 8635.
2. The Kepler Problem in Universal Variables

As it is well known, the relative two body problem is represented by the differential system

\[ \dot{r} = -\frac{\mu}{r^3} r, \]  

with \( \mu \) the Gaussian constant. This system has seven integrals (obviously, only six of them must be independent):

- the angular momentum vector \( c = r \times \dot{r} \)
- the Laplace vector \( A = c \times \dot{r} - \frac{\mu}{r} \)
- the energy \( h = \frac{1}{2} \dot{r} \cdot \dot{r} - \frac{\mu}{r} \).

Depending on the sign of the energy \( h \) (or equivalently on the value of \( ||A|| \), that is the eccentricity) the orbit is either elliptic \( (h < 0) \), or parabolic \( (h = 0) \) or hyperbolic \( (h > 0) \).

Equation (1) is singular for \( r = 0 \). Among the several ways in avoiding this singularity, we follow the one given in [4, 5]. By making two changes of the independent variables \( dt = r \, ds \) and \( ds = r \, d\phi \), and by choosing \( s_0 = \phi_0 = 0 \) at the initial time \( t_0 = 0 \), the system (1) is transformed into

\[ \frac{d^2 r}{ds^2} - 2hr = \mu, \]  

\[ \frac{du}{d\phi} = c \times u, \text{ with } u = \frac{r}{r}. \]  

This system is no singular at any point, and besides, the equations are linear with constant coefficients, which integration is immediate. Indeed, let us denote by

\[ x_1(s; h) = \frac{1}{2} \left[ \exp(\sqrt{2hs}) + \exp(-\sqrt{2hs}) \right], \]  

\[ x_2(s; h) = \frac{1}{2\sqrt{2h}} \left[ \exp(\sqrt{2hs}) - \exp(-\sqrt{2hs}) \right] \]

two particular independent solutions – its Wronskian is the unit – of the second order homogeneous Equation (2) (note that the function \( x_2 \) is regular for whatever value of \( h \) since \( \lim_{h \to 0} x_2(s; h) = s \)), and let us take the particular solution of the complete equation (2):

\[ x_3(s; h) = \frac{\mu}{2h} \frac{x_1(s; h) - 1}{x_2(s; h)}, \]

that also is regular for whatever value of \( h \). The general solution of (2) is the linear combination