GRAVITATIONAL INSTABILITY OF INTERSTELLAR MAGNETIC CLOUDS

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(Received 3 June, 1985)

Abstract. The gravitational instability of a nonrotating isothermal gaseous disk permeated by a uniform frozen-in magnetic field is investigated using a fourth-order perturbation technique. From the results it is found that the disk is stable when \( n/B_0 < \left( \frac{\pi^3 G}{3} \right)^{-1/2} \), where \( n \) and \( B \) are the column density of the disk and unperturbed magnetic field, respectively, and \( G \) is the gravitational constant. The disk is gravitationally unstable only when \( n/B_0 > \left( \frac{\pi^3 G}{3} \right)^{-1/2} \).

1. Introduction

Elmegreen and Lada (1977) have discussed in detail the observational evidence for the occurrence of shock-induced star formation. Based on the observational results, which seem fairly convincing, they have discussed the qualitative features expected for such a process occurring in the shocked layer of gas preceding an H II region as the region expands into a molecular cloud. They extended the theory by modelling layers of shocked gas as self-gravitating, pressure-bound, isothermal, plane-parallel gas sheets, and then studied the dispersion relation for unstable perturbations of such sheets. Welter and Schmid-Burgk (1981) performed similar calculations for the case of curved sheet geometry. However, Elmegreen and Lada (1977) and Welter and Schmid-Burgk (1981) left out from the investigation the effect of a magnetic field on the fragmentation processes of the layer.

Nakano and Nakamura (1978) considered the effect of a magnetic field on the fragmentation processes of the layer but to solve the problem they used a first-order perturbation theory which gives an error of 13% (Nakano, 1981).

Thus the whole problem of gravitational instability of a gaseous layer threaded by a magnetic field is still interesting with respect to star formation. This paper presents the results of the above-mentioned problem solved by use of fourth-order perturbation technique of Krylov-Bogoliubov-Mitropolsky as developed by Kakutani and Sugimoto (1974).

2. Nonlinear Schrödinger Equation

In this section the nonlinear Schrödinger equation has been derived from hydromagnetic equations which presents the dynamical behaviour of interstellar magnetic clouds. The unperturbed equations are (Spitzer, 1977)

\[ \rho \frac{\partial \vec{v}}{\partial t} = \frac{1}{c} (j \times B) - \nabla p - \rho \vec{\nabla} \phi, \]  

\[ \rho \frac{\partial \phi}{\partial t} = \frac{1}{c} \vec{j} \times B. \]
\[
\frac{\partial \rho}{\partial t} + \nabla (\rho v) = 0, 
\]

(2)

\[
\nabla^2 \phi = 4\pi G \rho, 
\]

(3)

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B), 
\]

(4)

\[
j = \frac{c}{4\pi} \left( \nabla \times B \right); 
\]

(5)

where \(v, \rho, \phi, \) and \(B\) are fluid velocity of the gas, density, gravitational potential, and magnetic field, respectively. The Cartesian coordinates \((x, y, z)\) are used in this paper. The gaseous layer is assumed to be at rest in the unperturbed state, its density is uniform in the \(x\) and \(y\) directions, and threaded by a uniform magnetic field in the \(z\) direction, \(B = (0, 0, B)\), which is perpendicular to the galactic plane and the fluid velocity streaming also along the \(z\) direction.

It is assumed that the gaseous layer will remain isothermal in the unperturbed and perturbed states. Now neglecting the radiation pressure, the gas pressure may be written as

\[
p = C_s^2 \rho, 
\]

(6)

where \(C_s\) is the isothermal sound speed. The density distribution in the unperturbed state is

\[
\rho(z) = \rho_0 \text{sech}(az), 
\]

(7)

where \(\rho_0\) is the density at the mid-plane \(z = 0\) and

\[
a = (2\pi G \rho_0)^{1/2}. 
\]

(8)

For weakly nonlinear systems, we can use the following expansion,

\[
\begin{bmatrix}
\rho \\
v \\
B \\
\phi
\end{bmatrix} = \begin{bmatrix}
\rho_0 \\
v_0 \\
B_0 \\
\phi_0
\end{bmatrix} + \varepsilon \begin{bmatrix}
\rho^{(1)} \\
v^{(1)} \\
B^{(1)} \\
\phi^{(1)}
\end{bmatrix} + \varepsilon^2 \begin{bmatrix}
\rho^{(2)} \\
v^{(2)} \\
B^{(2)} \\
\phi^{(2)}
\end{bmatrix} + \varepsilon^3 \begin{bmatrix}
\rho^{(3)} \\
v^{(3)} \\
B^{(3)} \\
\phi^{(3)}
\end{bmatrix} + \varepsilon^4 \begin{bmatrix}
\rho^{(4)} \\
v^{(4)} \\
B^{(4)} \\
\phi^{(4)}
\end{bmatrix}
\]

(9)

where 0 indicates the unperturbed state values.

The monochromatic plane wave is given by

\[
\rho^{(1)} = q \exp(i\psi) + \bar{q} \exp(-i\psi), 
\]

(10)

where \(q\) is the amplitude, \(\bar{q}\) is its complex conjugate, \(\psi = (kx - \omega t)\) is the phase, \(k\) is the wavenumber, and \(\omega\) the frequency. The amplitude \(q\) is a slowly-varying function