Landé Factors of Collective Mode Multiplets in $^3$He-B and Coupling Strengths to Sound Waves

N. Schopohl and L. Tewordt

Abteilung für Theoretische Festkörperphysik, Universität Hamburg, Hamburg, Germany

(Received March 10, 1981)

In a previous paper we predicted a linear splitting of collective modes in $^3$He-B with magnetic field, in particular, a fivefold splitting of the $(8/5)^{1/2}\Delta$ mode. A fivefold splitting of about the same size has been recently observed by Avenel, Varoquaux, and Ebisawa for the new sound attenuation peak. In this paper all 18 collective modes are classified in terms of total angular momentum $J = 0, 1, 2,$ and $J_z$ by means of a unitary transformation to the nine eigenstates $|J, J_z\rangle$. The Landé factors for the field splitting of the real and imaginary mode triplets and quintuplets [including the $(12/5)^{1/2}\Delta$ mode quintuplet] are calculated for all temperatures. We also determine the sound dispersion relation. The angular dependence of the coupling constants of the $J = 2$ quintuplets is given by $|Y_{2,2j}(\theta)|$, in agreement with the results of Avenel, Varoquaux, and Ebisawa for the new mode. We find a coupling constant of the $(8/5)^{1/2}\Delta$ mode to sound which is an order of magnitude smaller than that given by Koch and Wölfle. By comparing this with the experimental strengths we conclude that the particle–hole asymmetry parameter is about four times larger than that derived from the free–particle density of states.

1. INTRODUCTION

The observed new collective mode in $^3$He-B has a frequency which is about 20% lower than the frequency $\omega = (8/5)^{1/2}\Delta$ of an order parameter collective mode that was found first by Nagai and independently in Ref. 4. Nevertheless there are good reasons now to believe that the observed mode corresponds to this $(8/5)^{1/2}\Delta$ mode. Maki has classified the 18 order parameter collective modes in $^3$He-B in terms of total angular momentum quantum numbers $J = 0, 1, 2$. The $(8/5)^{1/2}\Delta$ mode belongs to $J = 2$ and is therefore fivefold degenerate. In a previous paper (called I in the following) we have shown that in a magnetic field $H$ this eigenfrequency splits into five eigenfrequencies and that the

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splitting goes linearly with the field for not too high fields \((H \leq 1 \text{ kG})\). Recently Avenel \textit{et al.}\textsuperscript{7} observed that the new sound attenuation peak splits linearly with magnetic field into five components. The observed values for the splitting are in fair agreement with the theoretical "Landé factor" for the \(J = 2\) quintuplet of the \((8/5)^{1/2}\Delta\) mode. For \(T = 0\) this Landé factor agrees with ours.\textsuperscript{6} Additional evidence given in Ref. 7 for the existence of a \(J = 2\) quintuplet is the observation that the angular dependence of the intensity of the five lines follows to a large extent a \(|Y_{2,J_z}(\theta)|^{2}\) pattern. Here \(Y_{2,J_z}\) are the spherical harmonics of rank 2 and \(\theta\) is the angle between the wave vector \(\mathbf{q}\) of the sound wave and the magnetic field \(\mathbf{H}\). The experimental coupling strength of the new mode is smaller by a factor of about \(6 \times 10^{-4}\) than that of the well-known \((12/5)^{1/2}\Delta\) mode.\textsuperscript{1} This has been explained by Koch and Wölfle\textsuperscript{8} as an effect of particle–hole asymmetry, which gives rise to a small coupling of the \((8/5)^{1/2}\Delta\) mode to a sound wave. Reasons have also been given in Ref. 8 as to why strong coupling effects or admixture of \(l = 3\) pair fluctuations might lead to a lower value of the observed frequency of the new mode in comparison to the ideal value \(\omega = (8/5)^{1/2}\Delta\).

The purpose of this paper is twofold. Our first aim is to classify all order parameter collective modes in \(^3\text{He-B}\) in terms of total angular momentum quantum numbers \(J\) and \(J_z\) and to calculate the Landé factors of all multiplets in the linear field regime. Our second aim is to determine the strength and angular dependence of all coupling constants between order parameter collective modes and sound waves. In this paper we shall concentrate on the new phenomenon of linear field effects and we shall neglect the quadratic field effects also considered in \(I\) which arise from an ellipsoidal deformation of the energy gap with respect to field direction. The formulation in this paper is based on a general theory by Schopohl\textsuperscript{9} which takes into account all fluctuation components of the Landau molecular field and applies also to non-unitary states. A consistent vector notation analogous to that of the vector \(\mathbf{d}\) for the order parameter brings out the underlying physics much more clearly than the notation of \(I\).

Basic for our theory are the expressions for the \(2 \times 2\) matrix Green’s functions \(F\) and \(G\) in terms of the \(\mathbf{d}\) vector describing the order parameter and the vector \(\Omega\), which is proportional to \(\mathbf{H}\) and whose length is equal to the renormalized Larmor frequency. The origin of the field splitting of the multiplets can be traced to the fact that the anomalous propagator \(F = (F_0\sigma^0 + \mathbf{F} \cdot \sigma)\omega\) acquires a spin-singlet component \(F_0\) proportional to \(i\omega_n(\mathbf{H} \cdot \mathbf{d})\), while the normal propagator \(G = G_0\sigma^0 + \mathbf{G} \cdot \sigma\) acquires a magnetic part \(G\) proportional to \((\mathbf{H} \cdot \mathbf{d})\). Here \(\sigma^0\) is the \(2 \times 2\) unit matrix and \(\sigma^i\) are the Pauli matrices. For \(\mathbf{H}\) in the \(z\) direction we see that \(F_0\) and \(G\) are different from zero only if \(\mathbf{d}\) has a \(z\) component, which means that the \(S_z = 0\) triplet pairing component has to be different from zero. Since \(F_0\) is