EVIDENCE FOR A LOW-DENSITY INFLATIONARY
UNIVERSE?

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Abstract. The inflationary universe model predicts the density parameter \( \Omega_0 \) to be \( \sim 1.0 \) with the cosmological constant \( \Lambda_0 \) usually taken to be zero, whereas observational estimates give \( \Omega_0 \leq 0.2 \) and \( \Lambda_0 \sim 10^{-57} \text{ cm}^{-2} \). It was found, however, that the observed variation of angular diameter with redshift for extragalactic radio sources could be interpreted in terms of a low density universe with linear size evolution of the sources for either an inflationary model with \( \Lambda \neq 0 \) or an open model with \( \Lambda = 0 \).

1. Introduction

The inflationary model of the Universe (Guth, 1981), which involves a quasi-de Sitter phase of exponential expansion, has been widely accepted, largely because it solves the so-called 'horizon' and 'flatness' problems of explaining why the Universe is so homogeneous and isotropic and so flat while being so large and old. One result of inflation is that the density of the Universe would have been driven so close to the critical density that even at the present time the density parameter \( \Omega_0 \) should be \( \sim 1.0 \).

A value of \( \Omega_0 \geq 1.0 \) has also been desired from other considerations such as the instability of clusters of galaxies, and from the esthetic viewpoint, since the Universe would then be closed. However, most estimates of the density parameter based on observational evidence, e.g., the direct determination of the galactic density using the mean mass to light ratio (Fang et al., 1982), estimates based on the cosmic-virial theorem (Peebles, 1979; Geller and Davis, 1978), and the primordial abundances of the light elements (Audouze, 1984), result in values of \( \Omega \) much less than unity. For this reason it has been suggested that there is some 'missing mass', which may be in the form of dark matter, hot, warm, or cold (e.g., Faber, 1984). There is also observational evidence that some dark matter is present in the Universe, at least in galactic halos (Faber, 1984). Recently, Loh and Spillar (1986) have claimed observational evidence for a value of \( \Omega \sim 0.9 \), although the techniques used have been criticized (Silk, 1986).

The cosmological constant \( \Lambda \) which appears in Einstein's field equations as a gravitational repulsive term (if \( \Lambda \) is positive) is generally taken to be zero. In terms of quantum field theory, Zel'dovich (1968) interpreted \( \Lambda \) as the invariant vacuum self-energy

\[
\Lambda = \frac{8\pi G}{c^4} \varepsilon_v,
\]

where \( \varepsilon_v \) is the energy density of the vacuum. Guth's model of the Universe in fact
requires such a $\Lambda$ term as the driving term for the inflationary expansion (Sciama, 1983), but this term would need to be powers of ten larger than the value set by cosmological limits. However, in supersymmetric Grand Unified Theories (GUT's) the cosmological constant can be made to vanish (Nanopoulos, 1984), which is usually taken to be the case.

On the basis of recent estimates of the age of the Universe from cosmochronology, Klapdor and Grotz (1986) found that the cosmological constant at the present time ($\Lambda_0$) should have a value in the range $(4.7-19) \times 10^{-57}$ cm$^{-2}$ for a Universe with zero curvature as required by inflation. Sandage and Tammann (1984) had previously shown that $\Lambda_0$ must be non-zero for a high-density universe ($\Omega_0 = 1.0$) with a Hubble constant $H_0$ in the range $50-100$ km s$^{-1}$ Mpc$^{-1}$. Furthermore, they found that even for a low-density universe ($\Omega_0 = 0.12$) $\Lambda_0$ must be non-zero for values of $H_0$ greater than 50 km s$^{-1}$ Mpc$^{-1}$.

Peebles (1984) noted that for an inflationary scheme with zero curvature (i.e., curvature constant $k = 0$) there is a simple relation between $\Lambda_0$ and $\Omega_0$
\[ \Lambda_0 c^2 = 3(1 - \Omega_0)H_0^2. \] (2)

This relationship is consistent either with the observational estimates for $\Omega_0$ ($\Omega_0 \leq 0.1$ to 0.3) and for $\Lambda_0$ $(4.7 - 19 \times 10^{-57}$ cm$^{-2}$) or with the values usually taken for an inflationary universe model, i.e., $\Omega_0 = 1.0$ and $\Lambda_0 = 0$.

In the present paper we will assume that the observational estimates for $\Omega_0$ and $\Lambda_0$ are reasonable and consider whether an inflationary scheme can be reconciled with the observational estimates and with the observed variation of angular diameter with redshift for extragalactic radio sources. This approach was also taken by Peebles (1984) who showed that several observational tests were equally consistent with either an open universe with $\Lambda_0 = 0$ and $\Omega_0 \sim 0.1-0.3$ or an inflationary universe with $\Lambda_0$ small but non-zero ($\sim 10^{-57}$ cm$^{-2}$) and $\Omega_0 \sim 0.1-0.3$.

2. The Angular Diameter-Redshift Test

Radio observations of extragalactic sources seem to be consistent with the relation $\theta \propto z^{-1}$ (Miley, 1968, 1971; Wardle and Pottasch, 1977; Wills, 1979), which can, therefore, be taken as a reference curve with which to compare theoretical models. Any acceptable theory must reproduce this reference curve for the angular diameter-redshift relation.

The Euclidean curve ($\theta \propto z^{-1}$) is steeper than the variation expected for a standard rod in a homogeneous Friedmann universe and is usually explained by saying that there must be linear-size evolution of the form
\[ l = l_0(1 + z)^{-x}, \] (3)
where $1 < x < 2$ depending on $\Omega_0$ (Kapahi, 1975). This linear size evolution has now been fairly well established, at least for radio galaxies (e.g., Kapahi, 1985; Eales, 1985; Okoye and Onuora, 1986).