PERTURBED ORBITAL ELEMENTS OF CLOSE BINARY SYSTEMS DUE TO TIDAL LAG IN LONGITUDE

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Abstract. This paper deals with the perturbations which tidal lag in longitude can produce to the orbital elements of a close binary system. The expressions obtained for the six elements of the orbit have been presented as functions of the unperturbed true anomaly, measured from the periastron. Our study includes the effects produced by the second, third, and fourth tidal harmonic distortions. In order to save space these extremely lengthy equations are given in the compact form of summations, by means of Hansen coefficients. Various recurrence relations, which hold good for Hansen coefficients, are also presented. Finally, this paper includes a second-order approximation only for the secular terms of first-order approximation.

1. Introduction

Proximity effects in close binary systems - observable both photometrically and spectroscopically - arising by gravitational interaction between their components, furnish an important way to study the physical properties of the constituent stars. Of these phenomena, the most important and observationally more noticeable are the tides raised on each star by its companion.

The tidal waves will remain stationary and give rise to no motion relative to a system of coordinates rotating with each star (equilibrium tides) if:

1. The equator of the rotating configuration is coplanar with the relative orbit of the disturbing body,
2. the orbit itself is circular, and
3. the (constant) angular velocity of each star is identical with the Keplerian angular velocity of orbital revolution; in other words, rotation is synchronized to revolution.

A breakdown of any one of these conditions is bound to give rise to dynamical tides. The tidal waves will cease to be stationary and will move relative to a rotating system of coordinates in astrocentric longitude as well as in latitude. Binary systems, the components of which are regarded as consisting of heterogeneous viscous fluid and revolving around their common centre of gravity in eccentric orbits, or their equatorial planes of axial rotation being inclined at arbitrary angles to the orbital plane, are subjected to tidal lag.

In the case of stellar matter, the presence of viscosity must be taken for granted, and consequently the individual tidal bulges will no longer be oriented exactly towards the disturbing body, but will advance or lag behind it. This depends on whether the angular velocity of axial rotation of the respective component is greater or smaller than the instantaneous angular motion of its mate. This phenomenon can produce, in the case of photometric observations, asymmetry with respect to the moments of conjunction.

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Sterne (1941) was the first who pointed out that effects produced by the second-harmonic tidal distortion are tantamount to those produced by orbital eccentricity if the apsidal line of the respective orbit were parallel with the line-of-sight. Dynamical tides in close binary systems have been studied by Kopal (1968a, b) while secular perturbations of the orbital elements have been presented by the same author (Kopal, 1972). A further investigation, which includes both secular and periodic perturbations, has been given by Zafiropoulos and Zafiropoulos (1982).

2. Equations of the Problem

In close binary systems the material of each star is characterized by a finite degree of viscosity. In consequence, the tidal bulges raised by each component on its mate will be of dynamical nature. The radius-vector connecting the centres of the two stars will cease to represent the axis of symmetry of the respective configuration.

In order to study the perturbations of dynamical tides on the orbital elements we employ the following six independent variables, which specify the orbit of a binary system in space:

\[ \Omega = \text{longitude of the ascending node}, \]
\[ i = \text{inclination of the orbital plane}, \]
\[ a = \text{semi-major axis of the relative orbit}, \]
\[ e = \text{eccentricity of the orbit}, \]
\[ \omega = \text{longitude of the periastron measured from the ascending node}, \]
\[ \chi = \text{modified mean anomaly given by} \]
\[ \chi = \varepsilon - (\Omega + \omega), \quad (1) \]

where \( \varepsilon \) is the difference between the true anomaly of the periastron passage and the mean anomaly. The first two elements determine the position of orbital plane in space, while the rest of them specify the properties of the orbit.

Dynamical tides raised by each component on its mate will cause fluid motions in the interiors of both stars, in a system of coordinates rotating with each component, i.e., the individual fluid elements will be compelled to move relative to their neighbours in astrocentric longitude \( (\epsilon_i) \) as well as latitude \( (\eta_i) \). In what follows, let us consider only the tidal lag in longitude \( (\epsilon_i = 0, \text{for } i = 1, 2) \) and disregard the tidal lag in latitude \( (\eta_i = 0, \text{for } i = 1, 2) \). Tidal lag in latitude can be produced only if the equator of the respective component is inclined to its orbital plane. Tidal lag in longitude can be due to asynchronism between rotation and revolution, even though equators are co-planar to the orbit. This lag in longitude can be expressed by the relation (Kopal, 1978; p. 237)

\[ \epsilon_i = \frac{c_i}{r^2} \left( \omega_i r^2 - n a^2 \sqrt{1 - e^2} \right), \quad (2) \]

where the constants \( c_i \) can be estimated for various types of stars (Kopal, 1968b, c); \( \omega_i \)