The conditions on the relative frequencies of coincidence between the measurements on two physical systems are deduced, in the particular case of four different directions, from Kolmogorovian probability and the Gutkowski and Valdes-Franco computational method. These conditions are compared with those imposed by Bell’s inequality. It is proved that Bell’s inequality is a necessary but not a sufficient condition for local Kolmogorovian probability. The further assumptions to be added to Bell’s inequality, in order to prove the equivalence with local Kolmogorovian probability, are studied. The connection with the results obtained by other authors on the subject is discussed.

Key words: locality, Kolmogorovian probability, Bell’s inequality.

1. INTRODUCTION

In a recent paper, Gutkowski and Valdes-Franco [1] proposed a computational method to find the conditions on the relative frequencies of coincidence between the measurements on two correlated physical systems when a local Kolmogorovian model exists.

In Sec. 2 we give the results obtained by this method, in Sec. 3...
the results we obtained are compared with Bell’s inequality, in Sec. 4 we discuss some connection with the results obtained by other authors on the subject, and in Sec. 5 we draw our conclusions.

The theoretical importance of relative frequencies in coincidence experiments is closely linked with the problem of locality and separability in quantum mechanics.

Let us suppose, for example, that a source $S$ of properly excited atoms emits statistical ensemble of polarization-correlated photon couples. Two optical systems, placed on opposite sides of the source $S$, collect part of the emitted photons and send them to two measuring apparatuses (Fig. 1). Each measuring apparatus consists of a polarizer (analyzer $A_k$) followed by two photomultipliers (detectors). The orientation $x_k$ ($k = 1, 2$) of analyzers $A_k$ can be varied at will. The analyzers decompose the incident light into an ordinary ray (polarized along the $x_k$ direction), detected by the photomultiplier $D_k(x_x)$, and an extraordinary ray (polarized along the $x_k$-orthogonal direction $y_k$), detected by photomultiplier $D_k(y_k)$.

By convention we say that the event $a(x_1) = +1 \ [b(x_2) = +1]$ occurred if a photon was detected by the photomultiplier $D_1(x_1) \ [D_2(x_2)]$ and $a(x_1) = -1 \ [b(x_2) = -1]$ occurred if a photon was detected by the photomultiplier $D_1(y_1) \ [D_2(y_2)]$.

Let $N(x_1, x_2; h, k)$ be the number of times that the “coincidence” event $a(x_1) = h, b(x_2) = k$ is observed; as the events $a(x_1) = h \land b(x_2) = k \ (h = -1, +1; k = -1, +1)$ are mutually exclusive events, the total number of coincidences is

$$N(x_1, x_2) = \sum_{h,k} N(x_1, x_2; h, k) \quad (1.1)$$

and the relative frequencies are defined, for $N(x_1, x_2) > 0$, by

$$R(x_1, x_2; h, k) = \frac{N(x_1, x_2; h, k)}{N(x_1, x_2)}. \quad (1.2)$$

The correlation function of two polarization measurements is given, as a function of $R(x_1, x_2; h, k)$, by

$$P(x_1, x_2) = \sum_{h,k} h k R(x_1, x_2; h, k). \quad (1.3)$$

For some atomic systems the polarization state of the photon-couple, according to quantum mechanics and neglecting the depolarization due to the finite solid angle of photon collecting optics, is

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left\{ |\widehat{x}_1\rangle |\widehat{x}_2\rangle + |\widehat{y}_1\rangle |\widehat{y}_2\rangle \right\}, \quad (1.4)$$