Viscous Fluid Universe Interacting with Scalar Field

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Taking a spherically symmetric isotropic line element, the case of a viscous fluid distribution interacting with scalar field is investigated. Four new solutions are obtained and the models are found to be expanding ones. Their physical and geometrical properties are studied.

1. INTRODUCTION

It is well known that meson particles with the charge of the electron and masses of the order of magnitude of 200 electron masses are found in cosmic rays. These particles have a good deal to do with the nuclear forces. The scalar meson field is a matter field and is associated with zero-spin chargeless particles such as \( \pi \) and \( \kappa \) mesons. The study of such a field in general relativity has been initiated to provide an understanding of the nature of space-time and the gravitational field associated with neutral elementary particles of zero spin. Scalar fields, as they help in explaining the creation of matter in cosmological theories, represent matter fields with spinless quanta and can describe the gravitational fields. Yukawa (1935) introduced the short-range meson field. Yukawa's theory is based on the assumption that all interactions must be transmitted through space from point to point by the mediation of a field, which is consistent with the principle of relativity; that is, the equations must be Lorentz-invariant.

Subsequently many authors took interest in the study of scalar fields. For example, Das (1962), Hyde (1963), and Das and Agarwal (1974) obtained solutions for the coupled gravitational and scalar fields. Rao et al. (1976) studied the interaction of a massive scalar field with a perfect fluid for the conformally flat, spherically symmetric metric. Banerjee and Santosh (1981), Froyland (1982), and Accioly et al. (1984) obtained different

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solutions for Einstein's field equations taking consideration of scalar fields. On the other hand, various authors (e.g., Heller et al., 1973; Heller and Klimek, 1975; Lukars, 1981; Maiti, 1982) studied the importance of viscous fluid from the cosmological solution point of view.

Here the motivation for taking the scalar field in addition to the viscous fluid as energy-momentum tensor is with a view to obtaining solutions for the cosmological model and to study its physical properties. It is noted that all the normal stresses are equal due to the spherical symmetry assumed and the shear viscosity factor drops from the field equations. The bulk viscosity need not be zero for the viscous fluid distribution interacting with the scalar field. The coefficient of bulk viscosity $\xi$ in the process of studying the solutions is found to be accompanied by a change in volume (that is, in density).

2. FIELD EQUATIONS

For this problem the line element considered is

$$ds^2 = e^\gamma dt^2 - e^\beta (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2)$$

where $\beta$ is a function of $r$ and $t$, and $\gamma$ is a function of $t$ only.

The energy-momentum tensor for a viscous fluid interacting with a massive scalar field is given by

$$T_{\mu\nu} = R_{\mu\nu} + S_{\mu\nu}$$

where $R_{\mu\nu}$ and $S_{\mu\nu}$ are, respectively, the energy-momentum tensors for the viscous fluid and the massive scalar field.

Here,

$$R_{\mu\nu} = \rho u_{\mu} u_{\nu} + (p - \xi \theta) H_{\mu\nu} - 2\eta \sigma_{\mu\nu}$$

where $p$ is the isotropic pressure, $\rho$ is the fluid density, $\xi$ and $\eta$ are the coefficients of bulk and shear viscosity, $\theta = u_{\mu}^\mu$ is the expansion factor of the fluid lines, $H_{\mu\nu}$ is the projection tensor defined by

$$H_{\mu\nu} = u_{\mu} u_{\nu} - g_{\mu\nu}$$

and $u_{\mu}$ is the flow vector satisfying the relation

$$g^{\mu\nu} u_{\mu} u_{\nu} = 1$$

In addition,

$$S_{\mu\nu} = \partial_{\nu} \varphi_{\mu} - \frac{1}{2} g_{\mu\nu} (\varphi_{\alpha} \varphi^{\alpha} - M^2 \varphi^2)$$