Particle Motion In Bell–Szekeres Space-Time

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We solve the geodesics equation for a charged particle in Bell–Szekeres space-time. In the same geometry we give the test particle solution of Dirac's equation.

1. INTRODUCTION

It is a well-known fact of classical electrodynamics in flat space that electromagnetic (e.m.) waves do not scatter, whereas in general relativity the nonlinear character is manifested by scattering of e.m. waves in analogy with photon–photon scattering of quantum electrodynamics. The space-time arising from collisions of shock e.m. waves was discovered by Bell and Szekeres (BS) (1974). This nonnull e.m. solution to the Einstein–Maxwell equations is characterized by nonsingular behavior in contrast to the Einstein solution resulting from the colliding gravitational plane waves (Szekeres, 1972; Halil, 1979). Another aspect of the BS solution is that off the wave front it is conformally flat, therefore by a theorem of Tariq and Tupper (1974) it must be transformable to a Bertotti–Robinson (BR) (Bertotti, 1959; Robinson, 1959) solution. This latter solution of Einstein–Maxwell equations is known to represent an e.m. radiation filled universe and is connected with the Reissner–Nordström “throat” which is defined (Misner et al., 1973) for the case of charge \((Q)\)=mass \((M)\) and where \(|Q−r|\ll Q\).

To our knowledge the solution of geodesics equations in BS geometry is absent and for BR is not without ambiguities (Lovelock, 1967) in the literature. From the cosmological point of view this problem is interesting since e.m. shocks produced by the astrophysical objects interact to develop BS regions. The only nonvanishing components of the e.m. field tensor

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admitted by the BS solution consist of $E_x = \text{const}$ and $B_y = \text{const}$. It is known from the motion of charged particles in conductors that in the presence of a constant external magnetic field a transverse potential arises (Hall effect). For the present case we can identify the Hall potential similarly and observe that the electric fields associated with this potential are collinear with $E_x$ but there is no chance that the two electric fields compensate each other.

We present the solution of geodesics equations in BS geometry and integrate the separable Hamilton–Jacobi functional completely. Since electron–positron pair creation is a frequently occurring phenomenon around pulsars, we investigate the solution of a Dirac particle in BS background. For this purpose we employ Chandrasekhar's (1976) treatment of Dirac’s equation in the test particle approximation.

2. GEODESICS IN BS SPACE-TIME

Let us consider the head-on collision of shock e.m. waves with constant profile and characterized by the null-tetrad scalars, $\phi_0 = F_{\mu \nu} l^\mu m^\nu = k^{1/2} b = \text{const}$ and $\phi_2 = F_{\mu \nu} \bar{m}^\mu n^\nu = k^{1/2} a = \text{const}$, respectively. Here $k = G/8c^4$ ($G =$ Newton's constant, $c =$ speed of light), $a$ and $b$ are real constants with our choice that $ab > 0$. For the detailed description of e.m. collision problem we refer to the article by Bell and Szekeres (1974).

If the null coordinates $u$ and $v$ represent the directions of propagations for e.m. shocks, we define new coordinates by

$$\xi = au + bv$$
$$\eta = au - bv$$

which will prove to be suitable in the sequel. The coordinate lines $\xi = \text{const}$ ($\eta = \text{const}$) represent families of elliptical (hyperbolic) curves. In these coordinates the BS solution is

$$ds^2 = \frac{1}{2ab} (d\xi^2 - d\eta^2) - \cos^2 \eta dx^2 - \cos^2 \xi dy^2$$

while the e.m. vector potential has a single surviving component,

$$A_x \equiv A = (2/k)^{1/2} \sin \eta$$

Note that the factor $1/2ab$ in the line element is not significant but for reasons of correspondence with the null coordinates we shall keep it. The