Dual Geometric Field Theory

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A Weyl geometry with a gauge-invariant, Riemannian subgeometry is used to geometrize the combined Einstein–Maxwell theory. A generalized Hamilton–Jacobi equation from particle mechanics emerges as an immediate consistency requirement. The time-independent, Coulomb field case is found to include at least lowest-order quantum effects as in wave mechanics. Possible microscopic entropy is identified.

1. INTRODUCTION

A straightforward generalization of existing Einstein–Maxwell theory into the gauge-invariant framework of the Weyl geometry (Weyl, 1922) is proposed. The broader kinematical framework of this geometry allows a geometric interpretation of the electromagnetic potential as well as the gravitational or metric field. However, the actual dynamical equations proposed by Weyl are not used; rather, a gauge-invariant metric tensor is constructed, and a Riemannian subgeometry is constructed on this. This subgeometry is gauge invariant, and allows an escape from the nonphysical effects of a pure Weyl geometry. It also allows the formulation of dynamical equations formally identical to those of the standard Einstein–Maxwell theory, but with two dimensionless constants. These equations will preclude the generation of further such subgeometries, leaving a dual-geometric description of the system. An immediate by-product of this is a scalar consistency relation, which the field quantities must satisfy also. This relation is a second-order, partial differential equation whose first-order term is of the form of the Hamilton–Jacobi equation of a charged particle in a combined gravitational and electromagnetic field. This will be identified as the equation of mechanics.

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The spherically symmetric solution to the Einstein–Maxwell equations is then examined. Spherical symmetry is not imposed on the new equation, though time independence will be assumed. Under these conditions, it separates into three ordinary differential equations that are further transformed into second-order linear equations. These will be seen to closely resemble the equations of the quantum theoretic, Coulomb-force problem. Indeed, the angular equations are identical. The radial equation is approximately the same, giving the same form in the nonrelativistic limit.

Closing sections then discuss constants, source motion, short-range effects, the physical identification of the new term in the mechanics equation in terms of entropy, and the concept of information as an exact, microscopic variable. The concept of “particles” is discussed also. Finally, the theory is contrasted with Weyl’s original theory.

2. KINEMATICS—THE WEYL GEOMETRY, GAUGE INVARIANCE, AND A SUBGEOMETRY

The geometry proposed by Hermann Weyl (1923) as a framework for a unified field is defined by a metric tensor, $g_{\mu\nu}, g\equiv\text{det}(g_{\mu\nu})\neq0$, and an intrinsic 4-vector, $v_\mu$. Together these determine an affine connection

$$\Gamma^\mu_{\nu\alpha} = \left\{ \begin{array}{c} \mu \\ \nu \alpha \end{array} \right\} + \delta^\nu_\alpha v_\alpha + \delta^\mu_\alpha v_\nu - g_{\nu\alpha} v^\mu$$

(1a)

where

$$\left\{ \begin{array}{c} \mu \\ \nu \alpha \end{array} \right\} = \frac{1}{2} g^{\mu\rho} (g_{\rho\alpha,\nu} + g_{\alpha\nu,\rho} - g_{\rho\nu,\alpha})$$

(1b)

A comma before a subscript denotes the partial derivative with respect to a coordinate; that is, $,\alpha = \partial / \partial x^\alpha$.

The quantity $\Gamma^\mu_{\nu\alpha}$ is invariant under the conformal-gauge transformation

$$\bar{g}_{\mu\nu} = s(x^\lambda) g_{\mu\nu}, \quad s \neq 0$$

(2a)

and

$$\bar{v}_\mu = v_\mu - \frac{1}{2} (\ln|s|)_{,\mu}$$

(2b)

We retain the absolute value for now, though it is common to restrict $s>0$. In the central-force example to be given, $s>0$ is used.