Electromagnetic Effects in Unified Field Theory

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In unified field theory we derive expressions for the electric current densities \( j \) and \( \rho \). We show that \( j \) and \( \rho \) depend on the intensities \( E \) and \( H \); \( E \) and \( H \) possess a common limit \( 1/\alpha \); and Coulomb’s law is not compatible with the unified theory.

1. PREPARATIONS

A. Einstein and B. Kaufman’s work (1954) on the eigenvalues of \( g_{ik} \) is our starting point. They introduce a system of reference connected with the “diagonal system” of \( g_{ik} \) (both are inertial systems) by a Lorentz transformation and give a matrix for \( g_{ik} \) in their system. E–K’s matrix is unfilled, which is inconvenient for our work.

In preparation, we first transform E–K’s matrix to a filled matrix of \( g_{ik} \) by a general Lorentz transformation. Next we find the contravariant components \( g^{ik} \) of our matrix, and connect \( g_{ik} \) with the intensities \( E \) and \( H \).

In the unified theory and in the inertial system moving relatively to E–K’s system with uniform velocity, we give a filled matrix for \( g_{ik} \) as

\[
(g_{ik}) = \begin{pmatrix}
1 & g_{12} & g_{13} & g_{14} \\
-g_{12} & 1 & g_{23} & g_{24} \\
-g_{13} & -g_{23} & 1 & g_{34} \\
-g_{14} & -g_{24} & -g_{34} & 1
\end{pmatrix} \equiv \begin{pmatrix}
1 & U_z & -U_y & -V_x \\
-U_z & 1 & U_x & -V_y \\
U_y & -U_x & 1 & -V_z \\
V_x & V_y & V_z & 1
\end{pmatrix}
\]

(1)

and

\[
U = U_x i + U_y j + U_z k, \quad V = V_x i + V_y j + V_z k,
\]
which is derived from E–K's matrix

\[
(g'_{ik}) = \begin{pmatrix}
1 & g'_{12} & 0 \\
-g'_{12} & 1 & 0 \\
0 & 1 & g'_{34} \\
\end{pmatrix}
\]

by a general Lorentz transformation.

Next, from (1), using \( U \) and \( V \) for brevity, we derive the contravariant components \( g^{ik} \) as

\[
g^{11} = \frac{1}{g} \left[ -1 + \mathbf{V}^2 - \left( \mathbf{U}_x^2 + \mathbf{V}_x^2 \right) \right], \quad g^{22} = \frac{1}{g} \left[ -1 + \mathbf{V}^2 - \left( \mathbf{U}_y^2 + \mathbf{V}_y^2 \right) \right]
\]

\[
g^{33} = \frac{1}{g} \left[ -1 + \mathbf{V}^2 - \left( \mathbf{U}_z^2 + \mathbf{V}_z^2 \right) \right], \quad g^{44} = \frac{1}{g} \left[ 1 + \mathbf{U}^2 \right]
\]

\[
g^{12}, g^{21} = \frac{1}{g} \left[ \mp \mathbf{U}_z \mp (\mathbf{U} \cdot \mathbf{V}) \mathbf{V}_z - (\mathbf{U}_x \mathbf{U}_y + \mathbf{V}_x \mathbf{V}_y) \right]
\]

\[
g^{31}, g^{13} = \frac{1}{g} \left[ \mp \mathbf{U}_y \mp (\mathbf{U} \cdot \mathbf{V}) \mathbf{V}_y - (\mathbf{U}_x \mathbf{U}_z + \mathbf{V}_x \mathbf{V}_z) \right]
\]

\[
g^{23}, g^{32} = \frac{1}{g} \left[ \mp \mathbf{U}_x \mp (\mathbf{U} \cdot \mathbf{V}) \mathbf{V}_x - (\mathbf{U}_y \mathbf{U}_z + \mathbf{V}_y \mathbf{V}_z) \right]
\]

\[
g^{14}, g^{41} = \frac{1}{g} \left[ \mp \mathbf{V}_z \mp (\mathbf{U} \cdot \mathbf{V}) \mathbf{U}_z + (\mathbf{U}_y \mathbf{V}_x - \mathbf{U}_x \mathbf{V}_y) \right]
\]

\[
g^{24}, g^{42} = \frac{1}{g} \left[ \mp \mathbf{V}_y \mp (\mathbf{U} \cdot \mathbf{V}) \mathbf{U}_y + (\mathbf{U}_x \mathbf{V}_z - \mathbf{U}_z \mathbf{V}_x) \right]
\]

\[
g^{34}, g^{43} = \frac{1}{g} \left[ \mp \mathbf{V}_x \mp (\mathbf{U} \cdot \mathbf{V}) \mathbf{U}_x + (\mathbf{U}_y \mathbf{V}_z - \mathbf{U}_z \mathbf{V}_y) \right]
\]

where

\[
g = \det(g_{ik}) = - \left[ 1 + (\mathbf{U}^2 - \mathbf{V}^2) - (\mathbf{U} \cdot \mathbf{V})^2 \right],
\]

Einstein defines the tensor density \( \alpha_{ik} = -g^{1/2}g_{ik} \) and suggests \( g^{ik} \) is the intensity Einstein (1955). From this and (2), we get

\[
\alpha E = g^{23} i + g^{31} j + g^{42} k = \frac{1}{(-g)^{1/2}} [\mathbf{U} - (\mathbf{U} \cdot \mathbf{V}) \mathbf{V}]
\]

\[
\alpha H = g^{14} i + g^{24} j + g^{34} k = \frac{1}{(-g)^{1/2}} [\mathbf{V} + (\mathbf{U} \cdot \mathbf{V}) \mathbf{U}]
\]