Complex Geometry, Unification, and Quantum Gravity. I. The Geometry of Elementary Particles

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The Poincaré group is replaced by $U(3, 2)$, the pseudounitary extension of the de Sitter group $SO(3, 2)$, as internal and space-time symmetries are combined in a geometric setting which invalidates the no-go theorems. A new model of elementary particles as vertical vectors on the principal fiber bundle $U(3, 2) \to U(3, 2)/U(3, 1) \times U(1)$ is introduced and their interactions via Lie bracket analyzed. The model accounts for the four known superselection rules: spin, electric charge, baryon number, and lepton number.

1. INTRODUCTION

For some time, many physicists have felt that a realization of Einstein's vision of a totally unified field theory would be necessary to provide a framework for elementary particle phenomena. The so-called "grand unified theories" (GUTs) attempted to explain the strong and weak nuclear forces and the electromagnetic force in terms of one fundamental force. The most successful of these theories was that of Georgi and Glashow (1974), based on the gauge theory of $SU(5)$. In spite of the initial success of this program, there were several fundamental problems. The GUTs do not attempt to describe gravitation and thus fall short of being totally unified theories. Also, most GUT-type theories lead to the possibility of proton decay and predict particles which have not been observed. The introduction of supersymmetries and "supergravity," designed to incorporate gravity and by-pass the "no-go" theorems, have only introduced more unobserved particles and thus widened the chasm between theory and observation. Consequently, a new program of particle interactions and quantization is needed and one will be introduced in this series of papers.

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The history of theoretical physics may be viewed as a search for conservation laws. Although the program had to be modified as research progressed, the original motivation was the observation that, within the Lagrangian formalism, conservation laws in turn imply the action of a continuous (Lie) group. Consequently, it seems that the search for a totally unified field theory is a search for the correct Lie group. The Lie group will characterize the conservation laws, and consequently, the geometry of the Lie group will characterize the interactions of the elementary particles as well. There are no conservation laws associated with the so-called “super-symmetries,” so they will not be used in the present work.

We begin a study of the elementary particles by describing their interactions, and conversely we begin a study of the fundamental forces by discovering those particles which interact via those forces. Any mathematical model of reality must begin with some assumptions about the way nature works. If we are to base our physics on group theory, it must lead to exact conservation laws: energy, momentum, angular momentum, spin, charge, baryon number, lepton number, etc.

The triumph of general relativity coupled with Einstein’s vision of a unified field theory, as well as the successes of the geometric approach to gauge theories, suggest that we should look for a model which includes gravitation (which means that it should be based on geometry, like general relativity), in which the forces are distinct, but ultimately are derived from the same geometry. The successes of gauge theories and of geometric quantization suggest that this geometry of elementary particle interactions should be based on the homogeneous space of some Lie group.

The geometry of elementary particles to be studied here has evolved from the geometric setting the author introduced several years ago (Love, 1984). The many reasons for looking at SU(3, 2) were discussed in that paper, but here the group SU(3, 2) is viewed as an extension of the well-studied de Sitter group SO(3, 2).

Once a group is chosen, there are many ways to obtain physics from the group. The prescription of the gauge theories runs into problems in the case of noncompact groups. But on a more fundamental level, a fatal flaw of the Lagrangian approach is the inevitable appearance of infinities when such a theory is quantized.

The Hamiltonian approach is essentially equivalent to the Lagrangian approach and will not be used as our starting point, although a “Hamiltonian” will appear in the final formulation of the theory. The standard way of introducing groups into physics is to look for the symmetries of the Lagrangian, the Hamiltonian, the S-matrix, or the space-time. The approach taken here is that the group SU(3, 2) is the fundamental object and