Inflation in Brans–Dicke Theory with Torsion

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We show that an inflationary phase may occur at a sufficiently early epoch in the Robertson–Walker universe model in Brans–Dicke theory with torsion. Some features of this inflationary scenario are briefly discussed.

1. INTRODUCTION

Inflation is important because it is thought that it can solve such cosmological problems as horizons, homogeneity and flatness, and the magnetic monopole (Abbott and Wise, 1984a–c). It is well known that in the RW universe model filled with an ideal fluid with pressure $p$ and energy density $\rho$, the scale factor $a(t)$ satisfies the Friedman equation

$$\ddot{a} = -4\pi(\rho + 3p)/3.\,$$

The dot denotes the derivative with respect to the cosmic time $t$. The inflationary condition $\ddot{a} > 0$ implies then the appearance of a negative effective pressure $p < -1/3$, which can only be achieved in this scenario with a dominant vacuum contribution to the total stress-energy tensor (Guth, 1981). Berman and Som (1989) showed that an inflationary phase may occur in the RW universe for the Euclidean case in Brans–Dicke theory with torsion, under the positive pressure condition. Gasperini (1986) showed that the spin density of the matter source can cause an inflation. The spin angular kinetic energy can also cause an inflation (Bradas, 1987). We will show that an inflationary epoch may occur in Brans–Dicke theory with torsion.

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2. FIELD EQUATIONS

We adopt the RW metric
\[ ds^2 = dt^2 - a^2(t)[dr^2/(1 - kr^2) + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2] \] (1)
where \( a(t) \) is a scale factor.

We introduce vierbein fields \( V_\mu^i \)
\[ V_\mu^0 = (1, 0, 0, 0), \quad V_\mu^1 = (0, a/(1 - kr^2)^{1/2}, 0, 0) \]
\[ V_\mu^2 = (0, 0, ar, 0), \quad V_\mu^3 = (0, 0, 0, ar \sin \theta) \] (2)
where the Latin letters denote the anholonomic coordinates and Greek letters denote the holonomic coordinates. Considering the 1-form \( \omega^i = V_\mu^i dx^\mu \), we have
\[ ds^2 = \eta_{ij} \omega^i \omega^j \]

Now we start with the following Lagrangian density for BD theory with torsion:
\[ \mathcal{L} = (-g)^{1/2}(-R\phi + \omega \phi_\mu \phi^\mu /\phi + 16\pi L_m) \] (3)
where \( R \) is the curvature scalar constructed from the connection \( \Gamma^k_{ij} \), \( \phi \) is the BD scalar field, and \( \phi_\mu = \partial_\mu \phi \); \( L_m \) is the Lagrangian density of matter. For a spinless Lagrangian, Jha et al. (1988) obtained the following equations:
\[ \phi G_{ij} = -(8\pi T_{ij} + \omega X_{ij}/\phi) \] (4)
\[ \phi T_{ik}^j = \phi_k \delta^j_i - \phi_j \delta^k_i \] (5)
\[ \Box \phi = 4\pi T/\omega \] (6)
where
\[ X_{ij} = \phi_i \phi_j - \frac{1}{2} \eta_{ij} \phi_k \phi^k \]
\[ T_{ij}^k = F_{ij}^k - F_j \delta_i^k + F_i \delta_j^k \]
\[ F_i = F_{ij}, \quad T = T^j_i \]
and \( F_{ij} \) are the torsion tensors, and \( T_{ij}^k \) are the modified torsion tensors. We consider an isotropic and homogeneous universe; then the scalar field is only dependent on time. From equation (5) we have the nonzero components of the torsion
\[ F_{10}^i = F_{20}^2 = F_{30}^3 = \dot{\phi}/2\phi \] (7)
For convenience, we denote \( h = \dot{\phi}/2\phi \).

From the first and second Cartan equations
\[ \frac{1}{2} F_{jk} \omega^j \wedge \omega^k = d\omega^i + \omega^i_j \wedge \omega^j \]
\[ d\omega^i_j + \omega^i_k \wedge \omega^j_k = \frac{1}{2} R_{jkf}^i \omega^k \wedge \omega^f \]