Fock-Type Representation of the Lie Superalgebra $A(0, 1)$

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A Fock space of two pairs of generalized creation and annihilation operators is constructed. These operators belong to the odd part of the Lie superalgebra $A(0, 1)$ and generate the whole algebra. The creation and annihilation operators define in the Fock space an infinite-dimensional irreducible representation of the algebra $A(0, 1)$.

In the present note we study one particular infinite-dimensional representation of the Lie superalgebra $A(0, 1)$ in the Kac notation (Kac, 1977). The method we use is similar to the one applied in the quantum theory of bosons and fermions. For instance, $n$ pairs of Fermi operators generate the Lie algebra $B_n$ of the group $SO(2n + 1)$. Therefore the Fock space of these operators determines an irreducible representation of $B_n$. In a similar way the Fock space of Bose or, more generally, of para-Bose operators defines a class of infinite-dimensional representations of the orthosymplectic Lie superalgebra (Gantchev and Palev, 1978). The operators we introduce are neither Bose nor Fermi operators. Their representation space, however, possesses all main features of the ordinary Fock space. In fact it is generated out of a vacuum vector by means of polynomials of creation operators. We were led to these operators in a search for some possible generalizations of the quantum statistics. The present paper is an investigation along this line. It should not be considered as an attempt to develop a representation theory for the Lie superalgebras. Our main purpose is to study the Fock space of the operators we introduce by the simplest available example, so that later

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on it will be possible to generalize the results to the case of several and even an infinite number of creation and annihilation operators.

The relations between the generators of the algebra $A(0, 1)$ can be derived through its three-dimensional exact representation. Denote as $e_{a\beta}$, $\alpha, \beta = -1, 0, 1$, a $3 \times 3$ matrix with 1 on the intersection of the $\alpha$th row and $\beta$th column and zero elsewhere. Let $L_0$ and $L_1$ be subspaces of $A(0, 1)$ with the basis written in the brackets, namely,

$$
L_0 = \text{lin. env. } \{e_{-1,-1} + e_{00}, e_{00} + e_{11}, e_{1,-1}, e_{-1,1}\}
$$

$$
L_1 = \text{lin. env. } \{e_{01}, e_{10}, e_{0,-1}, e_{-1,0}\}
$$

(1)

The multiplication $[,]$ in $A(0, 1)$ is defined as follows:

$$[[a, b]] = \{a, b\} \equiv ab + ba, \quad a, b \in L_1$$

$$[[a, b]] = [a, b] \equiv ab - ba, \quad a \text{ or } b \in L_0$$

(2)

and it is extended by linearity to the other elements.

In this case

$$A(0, 1) = L_0 + L_1$$

(3)

and $L_0, L_1$ are the even and odd part of $A(0, 1)$, respectively.

The representation-independent structure relations of the generators can be derived from (2) and the multiplicative law of the matrices $e_{a\beta}$,

$$e_{a\beta}e_{\gamma\delta} = \delta_{\gamma\delta}e_{a\beta}$$

(4)

Define the operators

$$A_1^+ = e_{10}, \quad A_1^- = e_{01}, \quad A_{-1}^+ = e_{-1,0}$$

(5)

These operators constitute a basis in $L_1$ and generate the whole algebra. Indeed, using (4) we obtain

$$\{A_1^+, A_1^-\} = e_{11} + e_{00}, \quad \{A_1^+, A_{-1}^+\} = -e_{1,-1}$$

$$\{A_{-1}^+, A_{-1}^-\} = -e_{00} - e_{-1,-1}, \quad \{A_{-1}^-, A_{-1}^-\} = e_{-1,1}$$

(6)

Let now $a_\eta^\xi, \xi, \eta = \pm$ or $\pm 1$, be the representation-independent generators of the Lie superalgebra $A(0, 1)$, corresponding to $A_n^\xi$. Using the equality (4) we find the following structure relations between the operators $a_n^\xi$:

$$[[a_\eta^\xi, a_\eta^\xi], a_\xi^\eta] = \eta \delta_\eta\eta a_\xi^\xi - \eta \delta_\xi\eta a_\xi^\eta$$

$$[[a_\eta^\xi, a_\eta^\xi], a_\xi^{-\eta}] = -\epsilon \delta_\xi\eta a_\xi^{-\eta} + \epsilon \delta_\xi\eta a_\xi^\eta$$

$$\{a_\eta^\xi, a_\eta^\xi\} = \{a_\xi^{-\eta}, a_\eta^{-\eta}\} = 0$$

(7)

In this notation

$$L_1 = \text{lin. env. } \{a_\eta^\xi | \xi, \eta = \pm\}$$

$$L_0 = \text{lin. env. } \{(a_\eta^\xi, a_\eta^\xi) | \xi, \eta = \pm\}$$

(8)

Throughout the paper $\xi, \eta, \epsilon = \pm$ or $\pm 1$; $[x, y] = xy - yx$ and $\{x, y\} = xy + yx$. 