The Dielectric Formalism for $^3$He–$^4$He Mixtures

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The microscopic theory of $^3$He–$^4$He mixtures with a Bose condensate is formulated in terms of the dielectric formalism. By expressing all correlation functions in terms of proper, irreducible contributions, one sets the stage for approximate calculations that will be consistent with various exact sum rules and Ward identities, just as in the case of pure $^4$He. The present analysis includes a symmetry-breaking term that allows us to deal with the continuity equations properly, and is valid at finite temperature. As a specific application, we express the normal fluid density $\rho_N$ in terms of the static $^4$He current–current correlation function. We also give the first formal proof that in the presence of a moving condensate, the $^3$He atoms make no direct contribution to the superfluid flow.

1. INTRODUCTION

In the microscopic theory of pure Bose liquids, the dielectric formalism first developed by Ma and Woo\(^1\) and Kondor and Szépfalusy\(^2\) has proved very useful. We recall that in the dielectric formalism, all Green's functions and correlation functions are expressed in terms of "regular" diagrams, i.e., those that are both "proper" (cannot be split by cutting a single particle line) and "irreducible" (cannot be split by cutting a single interaction line). This formalism has been developed extensively by Wong and Gould, with emphasis on either $T = 0$ K,\(^3\) or low temperatures.\(^4\) More recently, it has been used to discuss the properties of Bose-condensed liquids at arbitrary temperatures by Talbot and Griffin.\(^5,6\)

The great advantage of the dielectric formalism over other methods (such as reviewed in Section VI of Hohenberg and Martin\(^7\)) is that it exposes the dynamical consequences of Bose condensation in the structure of the various kinds of correlation functions. In particular, the density and longitudinal current correlation functions have "improper" parts that involve

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the single-particle Green's function. As a result, these correlation functions and the single-particle Green's functions share the same poles. This key feature of Bose-condensed systems is made most apparent in the dielectric formalism. Moreover, this formalism suggests well-defined approximation schemes starting from some set of diagrams for the regular functions that are consistent with various rigorous low-frequency sum rules and Ward identities.\(^5,^6\) As an example, the one-loop diagrams have been used to compute the dynamic structure factor \(S(Q, \omega)\) for a dilute, weakly interacting Bose gas at \(T = 0\) K by Bartley and Wong.\(^8\) More recently, the same approximation has been used\(^9\) to calculate \(S(Q, \omega)\) in superfluid \(^4\)He at high temperatures, where there is an appreciable number of thermally excited rotons.

The present paper is devoted to the extension\(^6\) of the dielectric formalism to a fully interacting system of Fermi and Bose particles, with due allowance for a Bose condensate. The example we shall concentrate on is \(^3\)He-\(^4\)He mixtures; more specifically, on concentrations of \(^3\)He and temperatures such that the two atomic species mix freely and form a superfluid phase. The \(^4\)He atoms form a Bose condensate and the resulting consequences in the various coupled correlation functions will be made most manifest by using a diagrammatic formulation in terms of irreducible, proper diagrams, i.e., the dielectric formalism. The main goal of this paper is to derive explicit expressions for various correlation functions of superfluid \(^3\)He-\(^4\)He mixtures in terms of regular contributions. Our results are rigorous and valid at all temperatures. As a specific application, we derive some exact relations using Ward identities analogous\(^3,^5\) to those for pure superfluid \(^4\)He. We relate the normal fluid density \(\rho_N\) to the long-wavelength static limit of the regular \(^4\)He longitudinal current correlation function. We also give a formal proof that in the presence of a moving condensate of \(^4\)He atoms, the \(^3\)He atoms do not directly contribute to the superfluid flow in spite of the \(^3\)He-\(^4\)He interactions.

Much of the theoretical and experimental literature on \(^3\)He-\(^4\)He mixtures is concerned with low concentrations and low temperatures (say, \(T \approx 0.6\) K). The emphasis has been on the properties of a dilute Fermi gas of quasiparticles. The region of higher temperatures, where there is an appreciable number of thermally excited phonons and rotons, has been largely ignored. We feel that the general formalism we set up in this paper provides a sound microscopic basis for model calculations on \(^3\)He-\(^4\)He mixtures, taking full account of the broken symmetry due to Bose condensation.

The phenomenological quasiparticle theory of \(^3\)He-\(^4\)He mixtures is discussed by Baym and Pethick,\(^10\) Ruvalds,\(^11\) and Ebner and Edwards,\(^12\) as well as in Chapter 24 of Khalatnikov's monograph.\(^13\)