Vortices in Inhomogeneous Superconductors with Periodically Modulated Structure

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A numerical solution of the London equation is presented, yielding the vortex field distribution in the presence of a periodic inhomogeneity. We find that the influence of modulation depends strongly on the period, vanishing when the vortex is extended over several periods. This “averaging” effect was not found in the earlier treatments.

1. INTRODUCTION

Type II superconductors with periodic structure modulation have recently attracted great interest, not only due to their unusual magnetic and transport properties,¹-³ but also because they can serve as model systems for the study of flux pinning⁴ and commensurate and incommensurate structures in two dimensions at zero temperature.⁵,⁶ Experimentally, such systems are prepared by depositing successive superconducting layers,¹ by spatial modulation of impurity concentration along the given direction in a superconducting alloy,⁶ or by thickness modulation of thin superconducting films.³ In all the above cases, the flux penetration is governed by accommodation of the (two-dimensional) vortex lattice to the (one-dimensional) periodic inhomogeneity modulation. Since the magnetic structure of each vortex (as well as their interaction) is perturbed, the first task in studying the flux penetration is to evaluate the field distribution of a single vortex as a function of its position with respect to modulation. This is the purpose of the present paper, in which a new method is adopted, different from earlier perturbative approaches.⁷,⁸ This allows us to work with periodic modulations of relatively large amplitude. On the other hand, we find that even for small modulation amplitudes the results differ from those given by the perturbation method, except in the local limit of large
modulation periods. In the case when the vortex size (of the order of the magnetic penetration depth) is comparable with the modulation period an “averaging” effect occurs, thus diminishing the influence of the inhomogeneity. This nonlocality effect cannot be obtained in the perturbative approach.

2. SOLUTION OF THE LONDON EQUATION

Considering individual vortices, we calculate their magnetic structure with the London model, valid for inhomogeneous superconductors with large average Ginzburg–Landau parameter $\lambda$ in the low- and intermediate-field regions. We describe the whole effect of periodic modulation (along a given direction, taken as the $x$ axis) by the corresponding variation of the magnetic penetration depth $\lambda = \lambda(x)$. Physically, this corresponds to situations$^{2,3}$ where the inhomogeneity locally changes the diffusion coefficient and thus the coherence length $\xi$ and the penetration depth $\lambda$.$^{9,10}$

Taking the external magnetic field perpendicular to the modulation axis (i.e., along the $z$ direction), we start with the London equation

$$H(r_0, r) + \text{curl} [\lambda^2(x) \text{curl} H(r_0, r)] = e_2 \phi_0 \delta(r - r_0) \quad (1)$$

which implies the flux quantization occurring in the inhomogeneous type II superconductors as well.$^7$ In the London gauge, Eq. (1) together with the boundary condition for the field vanishing at infinity, gives the field distribution $H(r_0, r)$ in the $xy$ plane for the vortex situated at $r_0 = (x_0, y_0)$.

The general method we are using is applicable for any even periodic function $\lambda(x)$, while the numerical calculations are performed for the case, relevant for modulated alloys,$^7,9$ when

$$\lambda(x) = \bar{\lambda}(1 + \delta \cos qx)^{1/2} \quad (2)$$

The modulation vector $q = (2\pi/L)e_x$, $\bar{\lambda}$ is the magnetic penetration depth of a homogeneous sample with the same average composition, and $\delta < 1$ is the amplitude of modulation. We note that the temperature dependence enters via $\bar{\lambda} = \bar{\lambda}(T)$.

Introducing the integral transformation

$$H(r_0, r) = -e_2 \frac{2\phi_0}{\pi q \bar{\lambda} \lambda(x_0) \lambda(x)} \int_0^\infty \mathcal{H} \left( \frac{q x_0}{2}, \frac{q x}{2}, \frac{s}{\bar{\lambda} / \bar{\lambda}} \right) \cos \left( \frac{y - y_0}{\bar{\lambda}} \right) ds \quad (3)$$

we get from Eq. (1)

$$\frac{d^2 \mathcal{H}(t_0, t; s)}{dt^2} - F(t, s) \mathcal{H}(t_0, t; s) = \delta(t - t_0) \quad (4)$$