Collective-Mode Spectroscopy of Textures in Superfluid $^3$He-B

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A phenomenological theory for the propagation of the real squashing modes in superfluid $^3$He-B is presented. This allows one to calculate the splitting of the real squashing (rsq) mode spectrum caused by the combined effects of magnetic field, dispersion, and texture in the experimentally important range of magnetic fields from 0 to $10^3$ Gauss. This serves to provide a tool for the rsq-mode spectroscopy of the $\hat{n}$-textures in $^3$He-B. In particular, a new "gyrosonic effect" is suggested: the intensity of the rsq modes generated in textures depends on the sense of rotation—even when the axis of rotation $\hat{\Omega}$ coincides with the direction of the ultrasound propagation $\hat{q}$.

1. INTRODUCTION

The multicomponent nature of the order parameter in superfluid $^3$He produces a multitude of textures, i.e., nonuniform distributions for the order-parameter field. These textures exist either as ground states under external forces, such as applied magnetic field, boundaries, superflow, and rotation of the vessel (which results in the lattice of quantized vortices) or persist as metastable objects whose large stability is brought about by topology, such as for solitons.

Until now, the most versatile tool for the investigation of the $^3$He textures has been NMR, which has yielded decisive information about the structures of solitons and vortices, and which revealed a phase transition between vortices (for a review, see Ref. 3). Another important tool, the ultrasonic technique, was used for the anisotropic superfluid $^3$He-A, where the attenuation of sound is very sensitive to the direction of the orbital anisotropy axis. Textural phase transitions were also found with this technique. However, this cannot be applied to the isotropic superfluid $^3$He-B.
The observation of an additional splitting of one of the $^3$He-B collective modes (for reviews on collective modes and ultrasound in superfluid $^3$He, see Halperin and Ketterson), i.e., the real squashing (rsq) mode, having the angular momentum $J = 2$ changed the situation: instead of the conventional fivefold ($=2J+1$) Zeeman splitting in an applied magnetic field, a sixfold splitting was observed. The additional doublet splitting of the central line—the one with $J_z = 0$—was interpreted in terms of the $^3$He-B order-parameter $\hat{n}$-texture, which makes it possible to probe the $^3$He-B texture with help of the rsq-mode spectroscopy.

The influence of textures on the splitting of the rsq modes has been investigated phenomenologically in the limit of large fields, when the Zeeman splitting exceeds the textural one. The validity of the phenomenology was confirmed within microscopic theory, which exploited the quasiclassical approach. The microscopic theory was recently further extended into the region of small fields.

However, we feel that the microscopic approach is too complicated to be practical for an interpretation of the actual experimental data. Moreover, the strong-coupling corrections have not been introduced in the approach of Ref. 13, and not all the Fermi-liquid parameters are sufficiently well known to produce the correct values for the parameters involved. On the other hand, the phenomenological approach may also be extended for the case of low fields—with only two phenomenological parameters involved; these characterize, respectively, the Zeeman splitting at high fields and the dispersion splitting in zero-field.

2. PHENOMENOLOGICAL EQUATIONS FOR THE rsq MODES

The collective modes in superfluid $^3$He-B are the oscillations of the order-parameter matrix $A_{\alpha i}$ near its equilibrium B-phase value, the orthogonal matrix $A_{\alpha i}^{(0)} = R_{\alpha i}$:

$$A_{\alpha i} = R_{\alpha k}(\delta_{ik} + u_{ik}), \quad u_{ik} \ll 1$$

The rsq mode, as the excitation having $J = 2$, may be described by a symmetric traceless matrix $u_{ik}$, which is real for the real squashing mode. The phenomenological Lagrangian for this propagating mode contains the following terms allowed by symmetry:

$$\mathcal{L} = -u_{ik}u_{ik} + \omega_0^2 u_{ik}u_{ik} + c_1^2 \nabla u_{ik} \nabla u_{ik} + c_2^2 \nabla u_{ik} \nabla u_{ik} + \varepsilon_{ijkl} \tilde{H}_{ij} u_{ik} u_{jk}$$

$$+ (\eta_{kkW} \nabla k u_{ik} + \eta_{SS} \omega \nabla k u_{ik}) \rho$$

Here $\omega_0$ is the rsq-mode frequency at zero wavevector ($q = 0$), the parameters $c_1^2$ and $c_2^2$ (in principle, $c_2^2$ may be negative; however, both theory and experiment suggest that it is positive and of the order of the Fermi velocity