Vortex Textures in $^3$He-A Near the Transition to the A$_1$ Phase

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The decay of the nonsingular doubly quantized vortex in $^3$He-A into a pair of Mermin-Ho vortex textures in the immediate vicinity of the transition to the A$_1$ phase is confirmed using the variational approach.

1. INTRODUCTION

In the past decade considerable efforts have been made to collect experimental information concerning the nature of vortices in the superfluid phase of liquid $^3$He. By means of NMR spectroscopy and ion “optics” vortex $\hat{I}$-textures of the type predicted by Seppälä and Volovik were discovered and explored in sufficient detail in the rotating $^3$He-A.

In the presence of a strong magnetic field and for moderate angular velocity $\Omega$ there appears a hierarchy of length scales

$$\xi_H \ll \xi_d \ll b_v = \left(\frac{\nu h}{2m_v\Omega}\right)^{1/2}$$

where $\xi_H$ is the magnetic healing length, $\xi_d$ denotes the dipolar healing length and $b_v$ is the radius of the Wigner-Seitz cell (carrying $\nu$ quanta of circulation) of the vortex lattice. In this situation around each vortex of the considered type an $\hat{I}$-textural soft core with dimensions of the order of $\xi_d \approx 10^{-3}$ cm is built up, this happening in the presence of the quasi-homogeneous background of the spin part of the order parameter (realizing the minimum of the magnetic anisotropy energy). The soft cores are the dipole-unlocked regions where localized spin excitations are formed. It is the detection of these trapped spin-waves which was the first manifestation of the presence of nonsingular vortex textures in the rotating $^3$He-A.

In the presence of the magnetic field (characterized by the dimensionless strength $h = \gamma H/kT$) a sequence of phase transitions is observed at $T_{c1} = (1 + \eta h)T_c$, and $T_{c2} = (1 - \eta h)T_c$, where $T_c$ is the zero field transition temperature from the normal state of liquid $^3$He to $^3$He-A and $\eta$ is the
Ambegaokar–Mermin particle-hole asymmetry parameter $\tilde{\eta} = (-\beta_{245}/\beta_s) \eta$, where $\beta_{ij...} = \beta_i + \beta_j + \cdots$ are coefficients in the free energy expansion with respect to the gap parameters $\Delta_1$ and $\Delta_2$. In the temperature interval $T_{c1} - T_{c2} = (\eta + \tilde{\eta}) hT_c$ the $A_1$ phase with Cooper pairing in the single spin configuration $\downarrow\downarrow$ is stabilized ($\Delta_2 \neq 0$, $\Delta_1 = 0$). Below $T_{c2}$ spin-up Cooper condensate with $\Delta_1 \neq 0$ appears and superfluid $A_2$ phase is formed.

The pronounced pressure dependences of $\eta$ and $r = -\beta_{245}/\beta_s$ are documented experimentally.\textsuperscript{7,8}

The structure of the soft cores of vortices in $^3$He-A$_2$ is formed as a result of the balance between the energy of inhomogeneity of the order parameter and the dipolar energy. In the vicinity of the continuous transition from the $A_2$ phase (which is a coherent mixture of Cooper condensates with spin configurations $\uparrow\uparrow$ and $\downarrow\downarrow$) into the $A_1$ phase (with the Cooper pairing in the $\downarrow\downarrow$ state only) the character of the dipole–dipole interaction is radically altered. This is easily understood by noting that a part of the dipolar energy is responsible for the Josephson coupling between the above-mentioned spin condensates. Therefore it becomes clear that near the $A_2 \rightarrow A_1$ phase transition one has to expect considerable deformation of the structure of the dipolar soft cores of the vortices.

2. STRUCTURE OF THE NONSINGULAR SV VORTEX

To proceed with the quantitative analysis of the structure of soft cores of vortices in the immediate vicinity of the $A_1$ phase we turn to the expression of the order parameter of $^3$He-A in the presence of a strong magnetic field:

$$A_{\mu i} = (\Delta/\sqrt{2})(\alpha_+ \hat{d}_i + i\alpha_- \hat{d}_2)_{\mu}(\hat{u}_1 + i\hat{u}_2)_i$$

where $\Delta^2 = (\Delta_2^2 + \Delta_1^2)/2$, $\alpha_\pm = (\Delta_\pm \pm \Delta_1)/2\Delta$. In (2) pairs of orthogonal unit vectors $(\hat{d}_1, \hat{d}_2)$ and $(\hat{u}_1, \hat{u}_2)$ define quantization axes $\hat{s} = \hat{d}_1 \times \hat{d}_2$ and $\hat{l} = \hat{u}_1 \times \hat{u}_2$ of spin and orbital angular moments of Cooper pairs, respectively.

For the superfluid state (2) the dipolar energy density is expressed by

$$F_d = -\frac{1}{2}\chi_N(\Omega_A/\gamma)^2[\alpha_+^2(\hat{d}_1 \hat{l})^2 + \alpha_-^2(\hat{d}_2 \hat{l})^2]$$

where $\chi_N$ is the magnetic susceptibility of the normal phase of liquid $^3$He, $\Omega_A$ denotes frequency of the longitudinal NMR and $\gamma$ is the gyromagnetic ratio for the $^3$He nucleus. Below we are going to consider the situation with $\mathbf{H} \parallel \Omega \parallel \hat{z}$. For a fixed homogeneous spin configuration with $\hat{d}_1 = \hat{x}$ and $\hat{d}_2 = \hat{y}$ (which corresponds to the minimum of the magnetic energy) we have

$$F_d = -\frac{1}{2}\chi_N(\Omega_A/\gamma)^2[\frac{1}{2}(1 + \beta)l_x^2 + \frac{1}{2}(1 - \beta)l_y^2]$$

where the Josephson coupling parameter $\beta = \Delta_1 \Delta_2 / \Delta^2$ gradually approaches zero as $A_2 \rightarrow A_1$ (for the $A_1$ phase $\beta = 0$).