EXACT BIANCHI TYPE-VI₀ VACUUM COSMOLOGICAL MODEL IN BRANS–DICKE THEORY

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Abstract. We present an exact solution of the vacuum Brans–Dicke field equations for cosmological models of Bianchi type-VI₀. The solution represents anisotropic universe which has no analogy in Einstein's theory.

1. Introduction

It has often been claimed that the Brans–Dicke (BD) field equations (1961) are more consistent with Mach's ideas. The scalar-tensor theories of gravitation gained new interest in cosmology by Dehnen and Obregón (1971, 1972) of exact solutions of these equations which has no analogy even for the larger values of the parameter ω. Recently, one of the authors obtained an exact solution of the vacuum BD field equations for a spatially homogeneous and anisotropic Bianchi type-I configuration and has shown the well-known Kasner universe as a special case (Shri Ram, 1983a). The same author also has obtained a plane symmetric vacuum Bianchi type-I cosmological model in this scalar-tensor theory of gravitation (Shri Ram, 1983b). These solutions have their analogues in Einstein's theory. However, since type-I models are a very special subset of spatially homogeneous models, one should consider more general Bianchi-type configurations to understand the dynamics of the Universe. Lorentz (1982) has presented an exact solution of the Einstein–Maxwell equations for cosmological models of Bianchi type-VI₀ which represents anisotropic universe with source-free electromagnetic field and perfect fluid matter.

In this paper we obtain an exact solution of the vacuum Brans–Dicke equations for the metric tensor of a Bianchi type-VI₀ configuration. The solution is found to have no analogue in Einstein's theory. The model has a point-type singularity and tends to anisotropic expansion.

2. Field Equations and Their Solutions

The metric of this class of model is

\[ ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2qx} dy^2 + C^2 e^{2qx} dz^2, \]  

(1)
where \( A, B, C \) are the cosmic-scale functions of time \( t \) and \( q \) is a non-zero constant. The coordinates \( x, y, z, t \) will be numbered as 1, 2, 3, 4 respectively.

The BD equations in vacuum (cf. Van den Bergh, 1980) are

\[
R_{\mu\nu} = -\frac{\omega}{\phi^2} \phi_{,\mu} \phi_{,\nu} - \frac{1}{\phi} \phi_{,\mu} \phi_{,\nu}, \tag{2}
\]

\[ \Box \phi = 0, \tag{3} \]

where the BD scalar \( \phi \) is the reciprocal of the gravitational constant. Other symbols have their usual meaning.

It then follows from Equations (1), (2), and (3) that the field equations to be considered are

\[
\frac{A_{44}}{A} + \left( \frac{A_4}{A} \right) \left( \frac{B_4}{B} \right) + \left( \frac{A_4}{A} \right) \left( \frac{C_4}{C} \right) - 2q^2 \frac{\phi_4}{\phi} \frac{A_4}{A^2} = - \left( \frac{\phi_4}{\phi} \right) \left( \frac{A_4}{A} \right), \tag{4}
\]

\[
\frac{B_{44}}{B} + \left( \frac{A_4}{A} \right) \left( \frac{B_4}{B} \right) + \left( \frac{B_4}{B} \right) \left( \frac{C_4}{C} \right) = - \left( \frac{\phi_4}{\phi} \right) \left( \frac{B_4}{B} \right), \tag{5}
\]

\[
\frac{C_{44}}{C} + \left( \frac{A_4}{A} \right) \left( \frac{C_4}{C} \right) + \left( \frac{B_4}{B} \right) \left( \frac{C_4}{C} \right) = - \left( \frac{\phi_4}{\phi} \right) \left( \frac{C_4}{C} \right), \tag{6}
\]

\[
\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} = -\omega \left( \frac{\phi_4}{\phi} \right)^2 - \frac{\phi_{44}}{\phi}, \tag{7}
\]

\[ q \left( \frac{B_4}{B} - \frac{C_4}{C} \right) = 0, \tag{8} \]

\[
\frac{\phi_{44}}{\phi} + \frac{\phi_4}{\phi} \left( \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) = 0; \tag{9}
\]

where the suffix 4 denotes differentiation with respect to \( t \).

It follows from Equation (8) that

\[ B = \mu C; \tag{10} \]

\( \mu \) being constant of integration. Without the loss of generality, we take \( \mu = 1 \).

If we introduce the new variables \( \alpha, \beta, \) and \( T \) by

\[ A = e^{\alpha}, \quad B = e^{\beta}, \quad dt = AB^2 \, dT, \tag{11} \]

the field equations (4)–(9) reduce to

\[ \alpha'' + \frac{\phi'}{\phi} \alpha' = 2q^2 e^{4\beta}, \tag{12} \]