ON FAST MAGNETOSONIC CORONAL PULSATIONS

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Abstract. The linear properties of the fast magnetosonic modes of a coronal loop modelled as a smooth density inhomogeneity in a uniform magnetic field are compared with the case of a step function slab. It is shown that the group velocity $C_g$ of the modes, important in determining the structure of impulsively excited wave packets, possesses a minimum for a wide class of profile including the slab, with the exception of the Epstein profile for which the minimum in $C_g$ moves out to infinity. Results for the simple step profile are thus of wider validity, and likely to be applicable to coronal loops.

1. Introduction

Fast magnetosonic waves may be trapped in regions of enhanced plasma density, corresponding to regions of depressed Alfvén speed (Roberts, 1981b; Edwin and Roberts, 1982, 1983; Cally, 1986), such as occur in coronal loops. The fast modes are strongly dispersive, the character of which is determined by the specific nature of the inhomogeneity. The propagation of fast magnetosonic waves in a plasma density inhomogeneity with smooth profile was considered by Edwin and Roberts (1988), for parabolic, exponential, and Epstein profiles. It was noted that the behaviour of the group velocity $C_g$ of a wave was different for different profiles. This circumstance is important for the formation of the characteristic signature of a pulsation in a coronal loop (Roberts, Edwin, and Benz, 1983, 1984; Murawski and Roberts, 1993, 1994).

Such pulsations have been invoked as an explanation for a variety of periodic and quasi-periodic events observed in the corona, principally at radio and X-ray wavelengths (see, for example, Aschwanden, 1987; Edwin, 1991, 1992; Bray et al., 1991).

We show here that the behaviour of $C_g$ is exceptional in the case of the Epstein profile and that the simple slab profile is in fact a good general guide to the behaviour of a smooth, sharply structured, profile.

2. Fast Modes of a Density Inhomogeneity

Consider the propagation of fast magnetosonic waves in a low-$\beta$ plasma inhomogeneity of density $\rho(x)$ stretched along the magnetic field $B_0 = B_0 \hat{z}$. Linear perturbations of the transversal plasma velocity $V_x = U(x) \exp(i\omega t - ikz)$ are described in the low-$\beta$ limit by the equation (Roberts, 1981a)

$$ \frac{d^2 U}{dx^2} + \left( \frac{\omega^2}{C_A^2(x)} - k^2 \right) U = 0, \tag{1} $$

where \( \omega \) is the frequency, \( k \) is the longitudinal wave number, and \( C_A(x) = \frac{B_0}{\sqrt{4\pi \rho(x)}} \) is the Alfvén speed. The characteristics of waves in the inhomogeneity are defined by the specific profile of \( C_A(x) \). If there is a hump of density, and so a depression in \( C_A \), then the inhomogeneity is a waveguide for fast waves (Edwin and Roberts, 1982). We consider waves trapped near the centre of the inhomogeneity, satisfying

\[
U(x \to \pm \infty) \to 0. \tag{2}
\]

Consider the density profile

\[
\rho(x) = \rho_\infty + (\rho_0 - \rho_\infty) \text{sech}^2[|x|/d]^p, \quad p \geq 1, \tag{3}
\]

which gives a density structure smoothly varying from \( \rho_0 \) at \( x = 0 \) to \( \rho_\infty \) as \( |x| \to \infty \), with the Alfvén speed correspondingly varying from \( C_{A0} \) at \( x = 0 \) to \( C_{A\infty} \) at infinity. For arbitrary power \( p \), the eigenvalue problem (1)-(3) requires a numerical solution. The profile (3) is of particular interest because it allows us to describe analytically two limiting cases: \( p \to \infty \) and \( p = 1 \).

When \( p \to \infty \), the density profile becomes the slab step function of width \( 2d \):

\[
\rho(x) = \begin{cases} 
\rho_0 & \text{if } |x| < d, \\
\rho_\infty & \text{if } |x| > d. 
\end{cases} \tag{4}
\]

We consider \textit{kink} modes, which have \( U \neq 0 \) at \( x = 0 \); \textit{sausage} modes (\( U(x = 0) = 0 \)) may be treated similarly. The transversal structure of trapped kink waves is oscillatory inside the slab (with \( U(x) \sim \cos n_0 x \)) and evanescent outside (\( U(x) \sim \exp[-|m_e x|] \)), where \( n_0^2 = (\omega^2/C_{A0}^2 - k^2) \) and \( m_e^2 = (k^2 - \omega^2/C_{A\infty}^2) \). The dispersion relation is determined by continuity of boundary displacements and the total pressure:

\[
n_0 \tan n_0 d = m_e. \tag{5}
\]

When \( p = 1 \), profile (3) becomes the well-known Epstein profile and Equation (1) reduces to

\[
\frac{d^2U}{dx^2} + \left( \frac{\omega^2}{C_{A\infty}^2} - k^2 + \frac{\omega^2}{C_{Ad}^2} \text{sech}^2(x/d) \right) U = 0, \tag{6}
\]

where \( C_{Ad} \) is the Alfvén speed based upon the excess density \( (\rho_0 - \rho_\infty) \). The kink mode with phase speed \( a = \omega/k \) is (see, e.g., Landau and Lifshitz, 1958; Adams, 1981)

\[
U(x) = \text{sech}^\nu(x/d), \quad \nu = \frac{|k|d}{C_{A\infty} \sqrt{C_{A\infty}^2 - a^2}}, \tag{7}
\]