The Growth Rate of $^3$He Crystals

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We have measured the relaxation of the shape of $^3$He crystals under the effect of gravity and surface tension, close to the minimum in their melting curve at $T_{min} = 0.32$ K. A growth rate is deduced, which is found to be at a maximum at $T_{min}$ when the latent heat is zero. We interpret this maximum value ($k = 0.18 \pm 0.04$ sec/m) as the intrinsic mobility of the liquid-solid interface and compare it with existing theories. We also consider on which side the latent heat is released during growth, and how it may cross the liquid-solid interface.

We report the first direct measurement of $k$, the intrinsic growth rate of $^3$He crystals or intrinsic mobility of the interface between liquid and solid $^3$He. When the temperature is homogeneous in the whole system, and if the surface is rough $^{1,2}$, $k$ is the ratio of the growth velocity $v$ to the difference in chemical potential $\Delta \mu = \mu_L - \mu_S$ between the liquid and the solid ($\mu$ is taken per unit mass). In ordinary systems, it is generally difficult to have access to this quantity. Indeed, when a pure crystal is grown from its melt, the growth velocity is mainly limited by the slow diffusion of the latent heat in the bulk phases on each side of the moving interface. However, the growth resistance associated with this diffusion may vanish if the heat conductivity is very large and the latent heat very small. This is the case in $^4$He below 1 K, where the growth rate of rough surfaces was shown to be very large by several experiments $^4$: the mobility of these $^4$He surfaces is only limited $^5$ by collisions with thermal excitations (phonons and rotons). In the case of $^3$He, the latent heat vanishes $^3$ at $T_{min} = 0.32$ K; Andreev and Parshin $^5$ suggested ten years ago that the growth rate $k$ should be about $m/p_F$, the $^3$He mass divided by the Fermi momentum of quasiparticles in the liquid.$^6$ More recently, Puech, Bonfais, and Castaing $^7$ obtained a lower bound for $k$ which was much larger than $m/p_F$. They explained this

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$t$Similar problems are encountered for the transmission of spin waves (Heff et al.) and for the mobility of the superfluid $^3$He A-B interface (Buchanan et al.); see Ref. 6.
discrepancy by supposing that the liquid quasiparticles interact with the crystal lattice more than the interface; their theoretical estimate is

\[ k = \left( \frac{3t}{4} \right) \left( \frac{m}{p_F} \right) \left( \frac{\rho_s \rho_L}{(\rho_s - \rho_L)^2} \right) \]

where \( \rho_L, \rho_S \) are the respective densities of liquid and solid \(^3\)He and \( t \) is the sticking probability of quasiparticles colliding with the crystal.

By looking at the shape of a pure single crystal of \(^3\)He, relaxing in some thirty seconds towards its equilibrium shape, we have measured the effective growth rate in a small temperature domain \((\pm 0.02 \text{ K})\) around \( T_{\text{min}} = 0.32 \text{ K} \). We observed that the growth is a linear function of the applied force. As expected, we found that the growth velocity (or the inverse relaxation time) goes through a maximum at \( T = T_{\text{min}} \) when the latent heat is zero. At this temperature, we interpret the ratio \( v/\Delta \mu = 0.18 \pm 0.04 \text{ sec/m} \) as the intrinsic growth rate \( k \). We compare it with existing theories and with the experiment by Puech et al.\(^7\). We also present a tentative interpretation of the measured increase in relaxation time on both sides of \( T_{\text{min}} \), when a nonzero latent heat \( L \) appears.

The experimental cell, where \(^3\)He crystals are grown, is immersed in a superfluid \(^4\)He bath which acts as a thermostat (Fig. 1). It is made of two parts. The lower one, where crystals are observed through sapphire windows, is a small cube \((3.5 \times 3.5 \times 5 \text{ mm}^3)\) where sintered copper was added for better thermalization. The upper part is a BeCu double membrane, which allows us to change the cell volume by changing the outside \(^4\)He pressure. This cell was originally built to measure the roughening temperature and surface tension of \(^3\)He crystals. It is described in more detail in another article.\(^2\) For our present purpose, it is only important to note that we use purified \(^3\)He which contains only 5 ppm \(^4\)He and that this residual \(^3\)He is presumably adsorbed as a small fraction of a monolayer on the sintered copper. Thus, differences in chemical potential due to concentration gradients may be neglected. (In previous experiments, where impure \(^3\)He with 200 ppm \(^4\)He was used, our crystals behaved very differently\(^1\)). At the equilibrium, the temperature is homogeneous within less than \( 1 \mu \text{ K} \) in our cell.\(^2\) Finally, crystal shapes (or values of chemical potentials) depend on surface tension, on the local pressure which itself depends on gravity, and on slight temperature differences which may occur during growth. Gravity \( g \) induces a hydrostatic equilibrium in the liquid; the surface tension \( \gamma \) relates the local difference in pressure \( \Delta P = P_L - P_s \) to the local curvature of the liquid-solid interface through the usual Laplace equation \( \Delta P = \gamma \left( 1/R_1 + 1/R_2 \right) \). We have assumed\(^2\) that the surface stiffness \( \gamma \) is isotropic, and therefore equal to the surface tension or free energy per unit area. Careful measurements\(^2\) of equilibrium crystal shapes gave us the value \( \gamma = 0.060 \pm 0.011 \text{ erg/cm}^2 \).