SCHRÖDINGER EQUATION AS AN OPERATOR
ACTING ON TEST FUNCTIONS
DESCRIBING OBSERVING SYSTEMS
AND THE EPR CORRELATION

Noboru Hokkyo

Hitachi Energy Research Laboratory
1168 Moriyamacho
Hitachi, 316 Japan

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A reformulation of quantum mechanics is introduced by regarding the Schrödinger equation \( E(f^+) = 0 \) for the retarded particle wave \( f^+ \) as an operator (functional) acting on the test function \( f^- \) satisfying the boundary conditions of the observing system: \( \langle E(f^+), f^- \rangle = 0 \). The variational expression for the transition amplitude of a particle between the particle source and the detector naturally arises in the dual space of the particle field and the test function. In the two-slit electron interference experiment, the test function plays the role of the “quantum potential” which carries the information of the detector and the slit locations backwards in time, while in the Einstein-Podolsky-Rosen process the test function describes the time reversed process of a pair of spatially separated fermions with arbitrarily chosen spin orientations progressing backwards in time to form a spherically symmetric compound state. The separation of the kinematics (spin correlation and the dynamics (spacetime aspect) of the EPR process is pointed out.

Key words: quantum mechanics, generalized functions, EPR correlation.

It is known that the Schrödinger’s differential wave equation is not well-defined at the source and the absorber loci where physical properties of the medium can change abruptly across certain spacetime boundaries. The generalized function (distribution or functional) [1], adumbrated by Heaviside in the 1890’s and pinpointed by Dirac
as the $\delta$ function, was defined not by its value at a point but by its action on continuous or merely locally integrable test functions.

The source-absorber interpretations of quantum mechanics by Wheeler-Feynman [3] and others [4] could also be criticized on the ground that there are no solutions to Schrödinger's equation that satisfy both initial and final conditions at the same time, so that the particle liberated from the source does not know where and when to be absorbed during its free flight.

In the present note a reformulation of quantum mechanics is proposed, which regards the Schrödinger equation as an equation for the functionals, satisfying generally discontinuous initial conditions, operating on the test function that satisfies final boundary conditions of the observing system. By being guided, so to speak, by the test function, the particle wave evolves in time as if directed towards the end of being caught by the detector distributions [4].

Let us regard the transition amplitude $\langle x_0, t_0 | x, t \rangle$ of a particle, described by the Hamiltonian $H$, between a pair of spacetime points, $(x_0, t_0)$ and $(x, t > t_0)$, as a functional acting on the test function $\{ z, t | x_1, t_1 \}$ at each spacetime point $(x, t)$ in a certain spatial domain $V$ for a time interval $t_0 \leq t \leq t_1$, and consider the generalized Schrödinger equation

$$\int_V dx \int_{t_0}^{t_1} dt \left[ \left( \frac{\partial}{\partial t} \frac{i}{\hbar} H \right) \langle x_0, t_0 | x, t \rangle \right] \langle x, t | x_1, t_1 \rangle = 0, \quad (1)$$

with

$$\langle x_0, t_0 | x, t \rangle = 0 \quad \text{for } t < t_0, \quad (2)$$

and

$$\langle x, t | x_1, t_1 \rangle = 0 \quad \text{for } t > t_1 \quad \text{and on } \partial V. \quad (3)$$

Then, if $\langle x_0, t_0 | x, t \rangle$ satisfies the retarded wave equation

$$\left( \frac{\partial}{\partial t} + \frac{i}{\hbar} H \right) \langle x_0, t_0 | x, t \rangle = 0, \quad (4)$$

the advanced wave equation

$$- \frac{\partial}{\partial t} \langle x, t | x_1, t_1 \rangle + \left\{ \langle x, t | \frac{i}{\hbar} H \rangle \right\} \langle x_1, t_1 \rangle = 0, \quad (5)$$

arises automatically from the functional equation (1) through integration by parts.

Let us next consider the integral expression

$$I = \int_V dx \int_{t_0}^{t_1} dt \left[ \left\{ \left( \frac{\partial}{\partial t} + \frac{i}{\hbar} H \right) \langle x_0, t_0 | x, t \rangle \right\} \langle x, t | x_1, t_1 \rangle \right. \left. - \left\{ S(x, t) \langle x, t | x_1, t_1 \rangle - U(x, t) \langle x_0, t_0 | x, t \rangle \right\} \right], \quad (6)$$