MODELLING THE FINE-STRUCTURE OF THE GEOID: METHODS, DATA REQUIREMENTS AND SOME RESULTS

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Abstract. The requirements for precise geoid models on local and regional scales have increased in recent years, primarily due to the ongoing developments in height determination by GPS on land, but also due to oceanographic requirements in using satellite altimetry for recovering dynamic sea-surface topography. Suitable methods for geoid computations from gravity data include Stokes integration, FFT methods, and least-squares collocation. Especially the FFT methods are efficient in handling large amounts of gravity data, and new variants of the methods taking earth curvature rigorously into account provide attractive methods for obtaining continental-scale, high-resolution geoid models. The accuracy of such models may be from 2–5 cm locally, to 50–100 cm on regional scales, depending on gravity data coverage, long wave-length gravity field errors, and datum problems. When approaching the cm-level geoid basic geoid definition questions (geoid or quasigeoid?) become very significant, especially in rugged areas.

In the paper the geoid modelling methods and problems are reviewed, and some investigations on local data requirements for cm-level geoid prediction are presented. Some actual results are presented from Scandinavia, where a recent regional high-resolution geoid model yields apparent accuracies of 2–10 cm over GPS baselines of 50 to 2000 km.

1. Introduction

In the eighties the fundamental geodetic problem of geoid determination has been revitalized due to an increased demand of geoid models for use in connection with satellite geodesy. As geoid data have become a direct observable with space techniques, on land through a combination of satellite positioning and levelling, and at sea through satellite altimetry, the geoid has seen more direct use as a primary data source in geophysics. For oceanography the geoid is required for separating dynamic sea surface topography from measured sea surface heights, and given the relatively small magnitude of the dynamic sea surface heights, independent geoid model requirements are high – often ±10 cm or less are accuracy level requirements.

In land areas regional or local geoid requirements are even higher, ±1 cm would be required if geoids were to be used in connection with differential GPS satellite positioning to fully replace conventional low-order levelling. The relationship of the geoid height $N$, GPS satellite determined ellipsoidal heights $h$, and conventional levelled heights $H$

$$h = H + N$$

forms the base not only for converting between $h$ and $H$ using a geoid model, but also for providing observed geoid values when $H$ and $h$ are known, i.e. for GPS on levelling benchmarks.

In an absolute sense geoid accuracies at $\pm 1$ to $\pm 10$ cm are currently impossible to attain. Accuracies in global spherical harmonic models (e.g. Rapp, 1990) are even in regions of the best data coverage hardly better than $\pm 50$ cm, a consequence of both systematic data errors, insufficient resolution, and uncertainties in the datum definitions. On local and regional scales, however, geoid models can be constrained to known geoid heights, and in a relative sense it then becomes feasible to talk of 1–10 cm geoids over ranges of 10–1000 km. This will be illustrated later in the paper by simulations and examples from Scandinavia, comparable results have been reported also by e.g. Denker (1990).

To obtain such high-resolution regional or continental-scale geoid models a necessary requirement is a good, dense gravity coverage. Such coverage is more likely on land than at sea, and land gravity data have higher accuracy than marine data. Highest geoid accuracies are therefore obtainable only in land areas, and this paper have been written mainly with land geoid applications in mind.

On land major practical and theoretical geoid prediction problems are associated with the topography. The topography has two effects: (1) It gives rise to a high-frequency gravity field "topographic noise" signal, and given the typical distribution of gravity stations this signal is typically undersampled yielding risk of aliasing, and, (2) The gravity data are sampled on a non-level surface, implying that method for computing geoid from gravity data must be capable of handling such a surface. This is not true for the classical Stokes formula.

The effect (1) may be diminished by using terrain reductions, i.e. subtracting a model of the topographic densities from the gravity data prior to geoid prediction, and later restoring the effects on the predicted geoid. In case (2) it becomes critical to examine not only the actual geoid prediction method used, but also which geoid type is actually predicted.

The classical geoid is defined as an equipotential surface inside the actual masses of the earth. This definition corresponds with the use of orthometric heights $H$. Many of the geoid prediction techniques actually give quasi-geoid heights $\zeta$ rather than the classical geoid. The quasigeoid may be viewed as the upward continued geoid to the terrain through the topographic masses. Both quantities obey Bruns' equation

$$N = \frac{T}{\gamma}, \quad \zeta = \frac{T}{\gamma}$$

(\(\gamma\) normal gravity) with the anomalous potential $T$ evaluated at sea level to the left, and at the surface of the topography to the right. Inside the topography $T$ is