TRANSFERS BETWEEN LIBRATION-POINT ORBITS IN THE
ELLIPTIC RESTRICTED PROBLEM *

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Abstract. A strategy is formulated to design optimal time-fixed impulsive transfers between three-
dimensional libration-point orbits in the vicinity of the interior \( L_1 \) libration point of the Sun-
Earth/Moon barycenter system. The adjoint equation in terms of rotating coordinates in the elliptic
restricted three-body problem is shown to be of a distinctly different form from that obtained in the
analysis of trajectories in the two-body problem. Also, the necessary conditions for a time-fixed two-
impulse transfer to be optimal are stated in terms of the primer vector. Primer vector theory is then
extended to non-optimal impulsive trajectories in order to establish a criterion whereby the addition of
an interior impulse reduces total fuel expenditure. The necessary conditions for the local optimality of
a transfer containing additional impulses are satisfied by requiring continuity of the Hamiltonian and
the derivative of the primer vector at all interior impulses. Determination of the location, orientation,
and magnitude of each additional impulse is accomplished by the unconstrained minimization of the
cost function using a multivariable search method. Results indicate that substantial savings in fuel
can be achieved by the addition of interior impulsive maneuvers on transfers between libration-point
orbits.

Key words: Primer vector, three-body problem, halo orbits.

1. Introduction

Trajectory design studies for scientific missions scheduled for launch in the 1990's
have investigated the use of three-dimensional libration-point orbits in the vicinity
of the interior \( L_1 \) libration point of the Sun-Earth/Moon barycenter system. With
interest in such trajectories increasing, efficient transfers between libration-point
orbits may offer more options and greater flexibility in trajectory design. Hence,
this research effort is directed toward the formulation of a strategy to design optimal
transfers between halo orbits near the \( L_1 \) libration point.

The problems of transferring a spacecraft between two points in a given gravita-
tional field such that the minimum amount of fuel is expended has been the subject
of numerous analyses; however, somewhat fewer results have been obtained for the
time-fixed case. Formulating the general time-fixed transfer problem as a problem
of Mayer in the calculus of variations, Lawden (1963) develops the necessary con-
ditions for optimal impulsive trajectories in a general gravitational field (position-
and time-dependent only) in terms of the primer vector. The term primer vector is
introduced by Lawden to denote the vector comprised of the adjoint variables
associated with the velocity.

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Lawden's necessary conditions are the cornerstone of primer vector theory which seeks to answer: (1) When will an additional interior impulse improve the performance of the reference solution? and (2) If an additional impulse is needed, where is it located and what is its magnitude and direction? Lawden states that, if the primer magnitude exceeds unity at any instant on a reference trajectory, then this path is non-optimal and its cost can be reduced. Also, Lion and Handelsman (1968) extend the utility of primer vector theory by considering non-optimal impulsive trajectories and establishing a criterion whereby an additional impulse accomplishes a decrease in the cost function. In an effort to answer the second question, Lion and Handelsman and Jezewski and Rozendaal (1968) establish methodologies for locating and orienting the additional impulse such that the greatest reduction in cost is realized. Additionally, utilizing a second-order approximation for the cost, Jezewski and Rozendaal determine an estimate for the impulse magnitude.

Several researchers employ primer vector theory as it applies to time-fixed trajectories in an inverse-square gravitational field in order to accomplish a specific mission objective such as rendezvous in the vicinity of a circular orbit (Prussing, 1969, 1970), multiple-impulse trajectories with inequality constraints (Jezewski and Faust, 1971; Jezewski, 1975), direct ascent rendezvous (Gross and Prussing, 1974), rendezvous between circular orbits (Prussing and Chiu, 1986), direct ascent interception (Prussing et al., 1989), or rendezvous and interception with path constraints (Taur et al., 1990). To date, transfers between two orbits in the three-body problem have been granted a modicum of consideration. D'Amario and Edelbaum (1974) investigate minimum-impulse transfers from a collinear libration point to circular orbits about the Moon. Utilizing a multi-conic technique to compute trajectories in the circular restricted problem, D'Amario and Edelbaum apply primer vector theory to construct and optimize time-fixed transfer trajectories. The consideration of transfers between halo orbits in the vicinity of the $L_1$ libration point was originally motivated by Pernicka’s (1990) examination of fuel-dumping transfers in conjunction with a potential near-term mission. One study performed by Gómez et al. (1991) uses manifold theory to determine a transfer between halo orbits in the circular restricted three-body problem.

The objectives of this study include the determination, in the elliptic restricted three-body problem, of time-fixed impulsive transfers between halo orbits near the $L_1$ libration point. Additionally, since the conservation of fuel is highly desirable, exploration of the optimality of a resultant transfer is paramount. Here, optimum is defined to be minimum characteristic velocity or, equivalently, minimum of the sum of impulsive maneuvers. The allowance for coastal arcs in the initial and final halo orbits is not investigated here. Only the addition of interior impulses is examined as a means of minimizing total fuel expenditure; consequently, a time-fixed optimal transfer between fixed endpoints results. If the transfer between the specified boundary conditions requires $K$ interior impulses to be optimal, then minimization of the total fuel expenditure compels $K$ successive minimizations of a function of four variables by some multivariable search technique. Since