Structure of the Condensate and Some Possible Ways to Observe It

J. W. Halley

School of Physics and Astronomy University of Minnesota, Minneapolis, MN 55455

We review some aspects of the experimental consequences both well known and proposed, of the existence of the single particle condensate in liquid helium four. We also extend these considerations briefly to pair correlations which microscopic calculations show to also exist in the fluid.

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1. INTRODUCTION

Superfluid helium four is the only boson superfluid known to occur in nature. It is interesting that, though the basic origin of the superfluidity, in a boson condensation, has been known for many years, experimental and theoretical details concerning the microscopic nature of the order parameter associated with the transition to the superfluid state remain sketchy. Formal relationships connecting the microscopic quantum description of the fluid with the two fluid hydrodynamics describing its flow were established many years ago. Nevertheless, there is no detailed theory which relates microscopic wavefunctions for the fluid (which are available) directly to a phenomenological description of the interesting and useful hydrodynamic properties of the superfluid. Calculations using these microscopic wave functions have also shown that the condensate has a richer geometrical structure than is generally realized, and includes pair correlations. These pair correlations can be shown to have a quantitative, but not a qualitative effect on the two fluid hydrodynamics.

Though the hydrodynamics and the short wavelength structure of superfluid helium have been very thoroughly studied experimentally, experiments which explicitly link the structure and magnitude of the condensate to experimental observables are quite rare. The well known deep inelastic neutron experiments to study the condensate are reviewed elsewhere. Here we focus on possible other probes which might reveal the existence and structure of the condensate in new and revealing ways.

In the next section we review the definition of the condensate including pair effects and computational estimates of the relevant magnitudes. In section three
we review some essential features of the connection to hydrodynamics following the treatment of Baym for the single particle component and filling in some details of an earlier paper concerning the relationship of a pair component of the condensate to the two fluid hydrodynamics. Section 4 discusses possible new experimental consequences of the existence of condensate correlations, emphasizing a proposed new experiment. Section 5 is a brief conclusion.

2. DEFINITION OF THE CONDENSATE

The hydrodynamics of superfluid $^4$He is described by the two-fluid model. The two fluid model was originally a phenomenological theory whose relation to the microscopic physics of liquid helium was established by Hohenberg and Martin. In the theory, the fluid is described in terms of the densities and velocity fields of two "fluids" denoted $n$ and $s$ together with the momentum flux density $\mathbf{f}$ and the local thermodynamic variables. It is important to define the meaning of variables $\varrho_n$, $\mathbf{v}_n$, $\mathbf{e}_n$ and $\varrho_s$ carefully, since intuition can be somewhat misleading. We can begin the discussion by defining the velocity superfluid velocity $\mathbf{v}_s$ in terms properties of the quantum mechanical wave functions of the fluid. At zero temperature, there is just one wave function, denoted $\Phi(\mathbf{r}_1, \ldots, \mathbf{r}_N)$ when there are $N$ helium atoms in the fluid. Fundamentally, superfluidity arises because of long range correlations in this wave function which may be expressed in terms of the one particle density matrix $\rho_1(\mathbf{r}, \mathbf{r}')$ which may be defined in terms of $\Phi(\mathbf{r}_1, \ldots, \mathbf{r}_N)$ as

$$\rho_1(\mathbf{r}, \mathbf{r}') = N \int d\mathbf{r}_2 \ldots d\mathbf{r}_N \Phi^*(\mathbf{r}, \mathbf{r}_2, \ldots, \mathbf{r}_N) \Phi(\mathbf{r}', \mathbf{r}_2, \ldots, \mathbf{r}_N)$$

In superfluid helium, $\rho_1(\mathbf{r}, \mathbf{r}')$ does not vanish as $\mathbf{r}$ and $\mathbf{r}'$ become very far apart:

$$\rho_1(\mathbf{r}, \mathbf{r}') \lim_{|\mathbf{r} - \mathbf{r}'| \to \infty} = \Psi_1^*(\mathbf{r}) \Psi_1(\mathbf{r}')$$

(More specifically, in terms of how things depend on the number of particles $N$ in the system, we mean that the right hand side approaches a function independent of $N$ as the number of particles becomes large and the separation $|\mathbf{r} - \mathbf{r}'|$ becomes large (in that order).) The function $\Psi_1(\mathbf{r})$ characterizes the long range quantum coherence of the many body wave function and is sometimes called the macroscopic wave function of the fluid. In general it can be written in the form:

$$\Psi_1(\mathbf{r}) = |\Psi_1(\mathbf{r})| e^{iS(\mathbf{r})}$$

that is, as an amplitude times a phase factor. It is easy to see that

$$\mathbf{v}_s \equiv \nabla S / \hbar m$$

has the dimensions of a velocity and it is quite intuitively plausible that a computation of the momentum of the fluid would show that a finite fraction of the particles in the fluid are effectively moving with that velocity. Because $\Psi_1$ expresses very long range properties of the wave function, it is also plausible that local perturbations would not affect it, so that the flows described by $\mathbf{v}_s$ are expected to be very stable and to lead to superfluidity.