THE SOLUBILITY OF $^{4}\text{He}$ IN $^{3}\text{He}$ BELOW 0.1 K

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The Zharkov-Silin Fermi Liquid theory of solutions of $^{4}\text{He}$ in non-superfluid liquid $^{3}\text{He}$ has been applied to the recent phase separation data of Nakamura et al. At zero pressure, the difference in binding between a $^{4}\text{He}$ atom in liquid $^{4}\text{He}$ and in liquid $^{3}\text{He}$ is smaller than previous estimates, and the $^{4}\text{He}$ effective mass is close to the bare mass. The volume measurements of Laheurte show that the difference in binding has a minimum near -11 atm. This implies an enhanced solubility of $^{4}\text{He}$ in $^{3}\text{He}$ below 0.1 K at this pressure, although there is experimental evidence that the solubility at 0 K remains zero.

Since the work described in our Poster has already been accepted for publication, we give only a summary, emphasizing the results about the solubility curve below 0.1 K, and adding a few details not included in the paper.

The solubility of $^{4}\text{He}$ in $^{3}\text{He}$ becomes exponentially small as $T \to 0$ and the Zharkov-Silin theory has traditionally been used to fit the measured solubility curve. The low temperature asymptotic form of the theory predicts that the solubility $X_4^*$ depends on temperature as

$$X_4^* = (T/T^*)^{3/2} \exp[-(E_{44} - E_{34} + \pi_3^4 v_4)/k_B T]. \quad (1)$$

Here $E_{44} - E_{34}$ is the difference in binding at 0 K for a $^{4}\text{He}$ atom in liquid $^{4}\text{He}$ and a $^{4}\text{He}$ in liquid $^{3}\text{He}$. The term $\pi_3^4 v_4$ is an almost negligible correction. The quantity $k_B T^*$ is
\[ k_B T^* = \frac{2\pi\hbar^2}{(m^*_4 v_3^2/3)}, \]  

where \( v_3 \) is the molar volume of \(^3\)He, and \( m^*_4 \) the effective mass of \(^4\)He quasiparticles in \(^3\)He. At zero pressure,

\[ T^* = (4.91 \text{ K}) \frac{m_4}{m^*_4} \]  

where \( m_4 \) is the bare \(^4\)He mass.

Solubility measurements by Laheurte\(^7\) gave 0.56 K for \((E_{44}-E_{34})/k_B\) and 4.5 for \(m^*_4/m_4\) at zero pressure. However, the data did not extend below 0.18 K. The Fermi Liquid theory for pure \(^3\)He is invalid above 0.1 K, and so the Zharkov-Silin extension of the theory to solutions should not hold outside this range. Recent low temperature measurements of the solubility by Nakamura et al.\(^8\) disagree with the theory as extrapolated\(^4\) from higher temperatures.

Another condition for the Zharkov-Silin theory to be valid is that the \(^4\)He quasiparticle states be well defined. This implies that the \(^4\)He scattering rate \( t^{-1} \) is small, \( \hbar t^{-1} \ll k_B T \). The diffusion constant, \( D = k_B T t/m_4 \) according to theory\(^2\), is measured to be \( -1.5 \times 10^{-5} \) at 0.5 K and the melting pressure\(^9\). This implies \( \hbar t^{-1}/k_B T \approx 10 \). However the \(^4\)He-\(^3\)He scattering amplitude in the Fermi Liquid region, calculated by Saam\(^3\), indicates that the scattering rate will become \( \approx 100 \) times smaller below 0.1 K, giving \( \hbar t^{-1}/k_B T \approx 0.1 \).

The new measurements of the solubility at zero pressure by Nakamura et al.\(^8\) extend down to 65 mK. Fitting the theory to their data, we find \((E_{44}-E_{34})/k_B = (0.21+0.03/-0.01) \text{ K}\) and \(m^*_4/m_4 = (1.1+0.4/-0.1)\). The data and fit are shown in Fig. 1.

The dependence of \( E_{44}-E_{34} \) on pressure is given by

\[ \delta(E_{44}-E_{34})/\delta P = v^*_4(P) - v_4(P). \]  

Here \( v^*_4 = [1+\alpha_0(P)] v_3(P) \) is the partial molar volume of \(^4\)He in solution at 0 K, where \( \alpha_0 \) is the "BBP" parameter. Values of \( \alpha \) measured by Laheurte\(^7\) at a high temperature, 1.2 K, are shown in the inset to Fig. 1.

Fig. 2 shows the results of integrating (4) using Laheurte's \( \alpha \), and assuming an uncertainty of \( \pm 0.04 \) in extrapolating the measured value of \( \alpha \) to low temperatures. The minimum in the curve near 11 atm suggests that the solubility of \(^4\)He in \(^3\)He below 0.1 K may be considerably enhanced compared to that at the saturated vapor pressure. It is unlikely that \( E_{44}-E_{34} \) becomes negative, since this