THE DEPARTMENT OF DEFENSE SELENODETIC CONTROL SYSTEM AND THE FORCE FUNCTION OF THE MOON

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Abstract. The figure of the moon as defined by the DOD-66 Selenodetic Control System is first studied. Then, using the derived equation for the surface and adopting the density law \( \delta = \delta_0 + \alpha \rho^p \), we have evaluated the volume integrals relating the form of the surface to the gravity harmonics.

1. A Study of the DOD-66 Selenodetic Control System

The recently published Selenodetic Control System of the Department of Defense (1967) most probably marks the end of efforts to determine the basic figure properties of the moon using earthbound optical or photographic observations. The new data that artificial lunar satellites have already started providing are expected to decrease the errors of the earthbound systems by as much as one complete order of magnitude, if not more. In addition, they will provide control points covering the remaining 41% of surface area that no earthbound system could cover.

The earthbound measurements are based entirely on the technique of stereoscopy, which is applicable by virtue of the optical librations of the moon permitting the terrestrial observer to view identifiable surface features from directions differing by at most 20°. This angle allows the determination of the absolute coordinates of point-size features of the central regions of the lunar disk with a spherical error a little less than half a kilometer (about 0.43 km), if we assume that the differential atmospheric refraction is allowed to introduce only 5 microns of circular error when measurements are made on photographic images of 17 cm in diameter. The above error is of the same order of magnitude as the differences between the axis of any reasonable ellipsoidal component of the lunar figure; and as a result, earthbased control systems can provide only qualitative arguments. To make things worse, only 59% of the surface can be covered by control points, and so all longitude-dependent harmonics of the figure escape estimation. However, because of the symmetric character of the zonal harmonics with respect to the \( Oyz \)-plane of the standard frame of reference, they can be somewhat approximated, provided their amplitudes are not below the 'noise' level of the measurements. But even when the noise does not exist, the zonals are subject to the uncertainty of the control points of the far side, although less so than the longitude-dependent harmonics, and this only by virtue of their definition.

At present, about one year or more since the first lunar satellite, the evidence available from the rough estimates of gravity harmonics suggests that in case of near

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homogeneity or radial dependence of the density distribution, the surface undulations of the moon, that can be described by harmonics of order less than or equal to 4, are of the order of 2 km (see e.g., Akim, 1966, or Michael et al., 1966). This implies that earthbound telescopes using the 20° angle differential for stereoscopic observations could provide approximations of the basic harmonics of the figure (including those of order and degree 4) good to within 25% of their value, if, of course, the far side could also be studied under similar conditions. Thus it appears that the cardinal contribution of the first generation lunar satellites will be the coverage of the entire globe by control points. The improvement in the accuracy of the coordinates of these points is only of secondary importance for the determination of the low order or degree surface harmonics. A good example of this situation is the marginal zone of the moon for which sufficient coverage has existed since the beginning of this century (Hayn, 1907); and earthbound observations have proven sufficient to establish the harmonics (Fourier terms) of the lunar profile, including terms up to order 4 (Carson et al., 1966). It should be pointed out here that there is remarkable agreement among measurements made over a span of more than sixty years. Although very little, if anything, can be concluded from this well-established result (see e.g., Goudas, 1965; Levin, 1967), it shows that earthbound observations covering the entire lunar globe, if possible, would have been sufficient to determine its basic figure harmonics.

Because it is impossible to establish a mean error for earthbound control systems, the evaluation of the latter becomes a complicated task. The main problem in applying the principle of stereoscopy to determine absolute coordinates of features is not the relatively small size of their stereo differential displacement, but rather the differential displacement caused by the anomalous refraction by the earth’s atmosphere which can be eliminated by statistical techniques only. If we determine the stereo-displacement of a feature using combinations of two out of n photographs exposed at different librations, the reduced coordinates of the feature will not be (excluding coincidence) the same in any two cases, because the effect of the anomalous refraction and the stereo-displacement are additive and inseparable. The former effect must be subtracted from the original measurement before any reduction is made. One way to ‘purify’ the measurements has been suggested in an earlier publication (Goudas, 1967). This suggestion was developed from a less rigorous but quite fruitful effort to eliminate the error in question applied earlier (ACIC, 1965).

As already mentioned, the task of evaluation of control systems becomes complicated when error criteria cannot be applied. Thus it is necessary to develop criteria of a different nature which depend on plausible statistical properties that a control system must fulfill in order to be acceptable. One such example is the sign of the coefficient of the second zonal harmonic of the figure of the moon, which has to be negative if there is any polar flattening. The dynamical polar flattening is a well-established fact for the case of the moon, and hence most likely a similar (qualitatively) flattening in the figure is expected to exist, by virtue of the fact that such a possibility is in harmony with the relevant theories and observations. If there is any flattening, the sign of the above coefficient must be negative. Indeed, if the moon acquired the