Physical Axiomatics: Freudenthal vs. Bunge

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The following remarks are intended to show that some of Freudenthal's recent criticisms of Bunge's Foundations of Physics are wide of the mark. Freudenthal sets his criticisms of detail in a framework of some general considerations of the role played by axiomatic theories in the foundations of physics. In particular, he considers the notion of the objects of an axiomatic theory, the relation of an axiomatic theory to reality, and the notion of the transformation group of a theory. These topics are considered below.

1. THE OBJECTS OF AN AXIOMATIC THEORY

Freudenthal, in his criticism of Bunge's work, contrasts two views. According to the first, exemplified in the history of geometrical axiom systems, the objects of, say, an axiom system of Euclidean geometry would be points, lines, planes, and so on. According to the second view, adopted by algebraists, the objects of an axiom system of Euclidean geometry would be Euclidean geometries, just as the objects of axiomatic group theory are groups. This second view is advocated by Freudenthal, and he therefore considers it desirable to write all axiom systems in the form of a definition of an explicit predicate. For example, he suggests that an axiomatic system of Euclidean geometry should start like, "A Euclidean geometry E is a system consisting of three sets..., and ... relations such that...," this being followed by an enumeration of the axioms. The explicit predicate in this case is the predicate "is a Euclidean geometry"; in group theory, the predicate would be "is a group." It is always possible to formulate an axiomatic theory in this way and there can be no
doubt that as far as pure mathematics is concerned this procedure is very convenient, and hence desirable; but it is not necessary. When one leaves the realm of pure mathematics, however, such a formulation is not particularly convenient and may even be misleading. In an application of an axiomatic theory, attention is generally restricted to one particular interpretation or model of the theory, and it is simply not true to say that group theory, interpreted in terms of, for example, the symmetries of a physical object, is concerned with groups as its objects. On the contrary, the objects of the interpreted theory are, in the example considered, the symmetries of the object in question, i.e., the elements of a particular group. Now, physical theories are always interpreted theories, not abstract mathematical theories, hence it is perfectly meaningful to speak of the objects of, for example, physical geometry as physical points, lines, and so on. Physical geometry is concerned with the one physical space, i.e., the set of physical points and its structure, not with any other spaces that might happen to satisfy the axioms of the theory in some other interpretation. But if physical geometry were formulated in some such fashion as, "A physical geometry is a system..., such that....," then, clearly, even if the axiom system were categorical, there would turn out to be a multitude of physical spaces, whereas physicists admit only one. A physicist asserts certain propositions, or axioms, about what he takes to be real; but he does not take these propositions as completely defining the piece of reality they are concerned with. Reality cannot be defined, it is there to be described and explained.

Bernays\(^{(a)}\) has drawn a distinction between what he calls "material" or "pertinent" axiomatics, and "descriptive" axiomatics. He gives as examples of the former, Newton’s mechanics and the presentation of thermodynamics by Clausius; of the latter, group theory, lattice theory, field theory (cf. also Hilbert and Bernays\(^{(a)}\)). Clearly, this distinction is essentially that drawn above between interpreted axiomatic theories and abstract theories. Freudenthal does not seem to recognize the existence of pertinent axiomatics, which is unfortunate since he is discussing, in Bunge’s formulations of the fundamental physical theories in *Foundation of Physics*, a collection of pertinent axiom systems. Freudenthal’s view is epitomized in his remarks “To work in one Euclidean geometry, in one plane projective geometry, one does not need an axiomatic superstructure; there the points and lines are still objects of the theory. Axiomatizing means that the original objects lose their status of object in favor of the geometry to which they belong.” Such a view seems to me to result from a confusion of axiomatization and formalization, in the sense of abstraction from the meaning of symbols in favor of purely syntactical or formal relations between them. Freudenthal’s view seems to imply that axiomatization is not necessary in physics, but it is surely formalization that is not required, since this would remove the essential tie between physics and reality. To sum up, the situation appears to be this: Abstract mathematical theories may be regarded as having as objects those structures which the theory defines, so that the objects of abstract group theory are groups; equivalently, one could say that the objects of an abstract theory are its models. On the other hand, a physical theory, being an interpreted theory, has no models, and its objects must be regarded as the designata of the nonlogical, or descriptive, terms of the theory. If this is granted, then it must be concluded that it is Freudenthal who has been misled as to the objects of physical theories, rather than Bunge.