Nonlinear Wave Mechanics and Particulate Self-Focusing

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A previous model for treating electromagnetic nonlinear wave systems is examined in the context of wave mechanics. It is shown that nonlinear wave mechanics implies harmonic generation of new quasiparticle wave functions, which are absent in linear systems. The phenomenon is interpreted in terms of pair (and higher order ensembles) coherence of the interacting particles. The implications are far-reaching, and the present approach might contribute toward a common basis for diverse physical phenomena involving nonlinearity. An intimate relationship connecting coherence, nonlocal interaction, and nonlinearity has been previously noticed in the physics of superconductivity. It is shown here that all these ingredients are consistently contained in the present formalism. The present theory may contribute to elucidate a controversial theory proposed by Panarella, who claims to have measured high-energy photons due to high-intensity laser radiation, which cannot be predicted on the basis of linear quantum theory. Panarella explains the new phenomena by stipulating a nonlinear intensity-dependent photon energy. It is argued here that nonlinearity, manifested in the presence of high intensity, may give rise to high- and low-energy photons, the so-called "effective" and "tired" photons, respectively. However, the present explanation does not involve ad hoc assumptions regarding the foundations of quantum theory. In analogy with the electrodynamic model, the present theory leads to particulate self-focusing in high-density streams of particles. Since such particulate beams are currently under consideration in connection with fusion reactions, this might be of future interest.

1. INTRODUCTORY REMARKS

Mutual influence between field theories in various branches of physics is very common. This is due to the fact that investigators are using similar

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mathematical tools. Often this leads to new insight into the physics of the pertinent fields of interest. Thus, presently a model for nonlinear electrodynamic wave systems is being investigated in connection with quantum mechanical systems. Recently a ray formalism was proposed for analyzing self-focusing of electromagnetic waves in lossless\(^1\) and absorbing\(^2\) media. The theory is based on an extended Fermat principle,\(^6\) and the transition to wave mechanics is straightforward. For linear absorbing media, the wave mechanical analog has been recently examined,\(^7\) showing the significance of complex potentials and giving a meaning to space- and time-independent probability density in dissipative systems. The similar procedure of investigating nonlinear wave mechanical systems, with reference to the electrodynamic model, pays even higher dividends, as shown below.

We start the next section with a formal representation of the weakly nonlinear wave mechanical system. A periodic solution is then assumed, facilitating the algebraization of the transformed equations. This yields a dispersion equation, similar to linear systems; however, here the amplitudes of the wave function are also involved in the dispersion equation.

The next section is concerned with the implications of the theory for various branches of physics. The relevance to the theory of superconductivity is pointed out. A somewhat tentative argument regarding "effective" and "tired" photons in high-intensity laser beams is given. This may contribute to the understanding of experiments cited by Panarella,\(^8\) without invoking his ad hoc modification of quantum theory.

Finally the problem of particulate self-focusing is considered, in analogy with the electrodynamic case.

2. GENERAL FORMALISM

A formalism is presented here for dealing with weakly nonlinear systems as defined below. The formalism is quite general, and no attempt is made to discuss special cases here. The method is the analog of the electromagnetic case discussed previously.\(^1\)\(^,\)\(^2\)\(^,\)\(^5\)

Consider a general wave mechanical system represented by

\[
L_{ij}\psi_j + a_{ij}\psi_j = 0
\]

where \(L_{ij}\) is a square matrix of operators involving space and time derivatives, \(a_{ij}\) is a square matrix whose entries may be space and time dependent, thus describing potentials, and \(\Psi = (\psi_1, \ldots, \psi_j, \ldots)\) is a vector of probability wave functions. The dimensionality of \(\Psi\) is determined by the quantum mechanical model at hand, e.g., for the scalar Schrödinger equation \(\Psi = (\psi_1)\), and for the Dirac wave equation\(^10\) \(\Psi = (\psi_1, \ldots, \psi_4)\).