Systematic Derivation of All the Inequalities of Einstein Locality

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We show that a method exists for deducing from Einstein locality all the possible inequalities for linear combinations of correlation functions. This allows us to show also that there is a complete observational equivalence of deterministic local theories and probabilistic local theories.

Einstein locality\(^1\) is fundamentally the hypothesis that the results of measurements on atomic systems are determined by "elements of reality" (sometimes called hidden variables), associated to the measured systems and/or to the measuring apparatus, which remain unaffected by measurements on other, distant atomic systems. This determination can be either deterministic\(^2\) in the true philosophical sense, or probabilistic,\(^3\) in the sense that only the probabilities of the different outcomes of correlated measurements are fixed by the hidden variables. In either case Einstein locality leads to several interesting empirical consequences, the best known of which is Bell's inequality.\(^4\) Several authors have in recent years deduced new inequalities from Einstein locality.\(^5\) Most of them turn out to be violated by quantum mechanics. Attempts at understanding such violations in physical terms have consistently failed.\(^6\) Even well-known nonlocal theories, such as Newtonian dynamics or the de Broglie–Bohm hidden variable models, are unable to violate the inequalities deduced from Einstein locality.\(^7\) This being the case, it is felt that only extreme remedies will be able to render these implications of quantum mechanics acceptable: proposals range from the discussion of special relativity\(^8\) to the idea that communications between

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\(^2\) Particularly promising seems to the line of research of Vigier, which has, however, not yet provided a complete physical basis for the quantum mechanical spin predictions.\(^6\)
two regions with spacelike separation can take place through a propagation of information toward the past. Since the experimental situation is as yet not as sharp as it should be in matters of principle of such an importance, owing particularly to the necessity of additional (doubtful) assumptions for comparison with the quantum mechanical predictions, it is perhaps possible to think, alternatively, that the quantum mechanical treatment of correlated distant atomic systems may require some change.

In all cases it is interesting to deduce systematically from Einstein locality all the possible inequalities which it can generate, particularly because they could be tested in the new experiments on quantum correlations.

In order to do so it is useful to refer to the probabilistic formulation of Einstein locality, since it is more general than the deterministic formulation, as will be seen later. One considers therefore two measurements in the spacetime regions $R_1$ and $R_2$ with a spacelike separation. In $R_1$ the first observer measures the dichotomic observable $A(a)$ dependent on the instrumental parameters $a$, while in $R_2$ the second observer measures the similar observable $B(b)$. These measurements are performed on correlated systems, for instance, on two photons produced in the same atomic cascade. The only possible values of the dichotomic observables are assumed to be $\pm 1$. In the probabilistic approach one introduces the probabilities $p_\pm(a\lambda)$ that the result of the measurement of $A(a)$ gives $\pm 1$, respectively, and the analogous probabilities $q_\pm(b\lambda)$ for $B(b)$. Here $\lambda$ is a single symbol for three different sets of variables: those contained only in $p_\pm$, those contained only in $q_\pm$, and those common to $p_\pm$ and $q_\pm$. The notation with a single $\lambda$ is used for simplicity, the generalization to several variables being straightforward.

Einstein locality is implicit in these preliminaries, being embodied in the lack of dependence of $p_\pm(a\lambda)$ on $b$ and of $q(b\lambda)$ on $a$. If $\rho(\lambda) \geq 0$ is the probability density of the variables $\lambda$, the four possible joint probabilities for the correlated measurements in $R_1$ and $R_2$ are

$$\omega_{\pm,\pm} = \int d\lambda \rho(\lambda) p_\pm(a\lambda) q_\pm(b\lambda)$$

where for all $\lambda$, $a$, and $b$ one has

$$p_+(a\lambda) + p_-(a\lambda) = 1$$

$$q_+(b\lambda) + q_-(b\lambda) = 1$$

One well-known additional hypothesis made in such cases is the following: counter efficiencies do not depend in any way on the presence and orientation of polarizers and/or on the hidden variables of the incoming photon.