Relativistic Dynamics of Stochastic Particles

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Particle motion in stochastic space, i.e., space whose coordinates consist of small, regular stochastic parts, is considered. A free particle in this space resembles a Brownian particle the motion of which is characterized by a dispersion $D$ dependent on the universal length $l$. It is shown that in the first approximation in the parameter $l$ the particle motion in an external force field is described by equations coincident in form with equations of stochastic mechanics due to Nelson, Kershow, and de la Pena-Auerbach. A method is proposed for the relativization of the scheme used to describe the processes in the stochastic space; by using this method, the equations of particle motion can be written in a covariant form.

Interest in stochastic processes and fields has grown in recent years. This is mainly due to the close correspondence between stochastic processes and quantum mechanics\(^{(1)}\) and Euclidean quantum field theory.\(^{(2)}\) Nelson’s stochastic quantization has been generalized to the case of continuous systems\(^{(3)}\) and also to the cases of particles with spin\(^{(4)}\) and relativistic mechanics.\(^{(5)}\)

There are also other approaches to the investigation of stochastic processes and fields. Some of them start with a hypothesis on the stochastic property of the electromagnetic vacuum\(^{(6)}\) (this approach is called stochastic electrodynamics). Other approaches are based, more or less, on the concept of stochastic spaces.\(^{(7)}\) One assumes in this case that the random behavior is caused by the stochastic character of physical space (in analogy with Brownian motion). In other words, the stochastic character of the processes is interpreted as being a result of the action of space alone on the considered physical system.

Following an idea of Blokhintsev,\(^{(7)}\) we investigate the problem of the motion of a particle whose coordinates in a stochastic space $R_4(\xi)$ are defined

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by two terms $\hat{x}_\mu = x_\mu + b_\mu$, where $x_\mu$ is the regular part of the coordinates and $b_\mu$ is random vector with a distribution $\tau(b_\mu)$ obeying the condition

$$\int d\tau(b_\mu) = 1, \quad d\tau(b_\mu) \geq 0$$

In the nonrelativistic case it is sufficient to assume the stochastic property of the space component $x_i = \hat{x}_i = x_i + b_i$, but in the relativistic case such an operation needs some explanation. The space $R_4(\delta)$ in a relativistic theory must be the Minkowski space. The indefiniteness of the metric of this space leads to specific problems which do not appear in the case of Euclidean space. These specific difficulties are connected with the invariance assumption and the normalization condition for the probability of a value of an interval in the indefinite-metric space. The invariance assumption, roughly speaking, means that the distribution $\tau(b_\mu)$ of the vector $b_\mu$ must be a function of the interval $b^2 = b_\mu b^\mu$, and the normalization condition gives the equality $\int d\tau(b,b') = 1$. These two conditions cannot be fulfilled simultaneously in the Minkowski space. However, one can get rid of the above difficulty by assuming that the stochasticity of the space $R_4(\delta)$ appears in a Euclidean region of the variables $\delta_\mu$. Using the language of random fluctuations, this means that the fluctuations appear in the Euclidean space $E_4(\delta)$. We make, at the same time with this assumption, a shift of the coordinates in such a way that the coordinate $x_0 = ct$ will obtain a pure-imaginary additive term, and the coordinates $x_i$ will stay real. This shift procedure is deeply connected to the fundamental problem of causality and it is also directly related to the relativistic-invariant description of extended objects. There also exists a correspondence of this procedure to the transformation to the pseudo-Euclidean region in the case when the construction of quantum field theory proceeds from the Euclidean picture (more details can be found in the monograph of Efimov). Thus, starting with a construction in a Euclidean stochastic space, the relativistic-invariant description of the particle motion in a stochastic space may be realized. The attraction of the approach based on the idea of the stochasticity of physical space is mainly due to the fact that it allows one to generalize the stochastic mechanics of Nelson to the relativistic case.

If we consider the problem of random movement within the framework of spatial stochasticity, then it can be shown easily that the probability that

\[ \text{In the nonrelativistic case, the space stochasticity makes it impossible to determine a space coordinate of the particle with an error less than a certain length, for example, the Compton length of this particle. In the relativistic case within our framework there are some difficulties connected with physical interpretation of this property. Formally, it can be interpreted as the presence of fluctuations of four-dimensional coordinates in the Euclidean space } E_4(\delta_\mu). \]