The Necessity of Quantizing the Gravitational Field

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The assumption that a classical gravitational field interacts with a quantum system is shown to lead to violations of either momentum conservation or the uncertainty principle, or to result in transmission of signals faster than c. A similar argument holds for the electromagnetic field.

1. INTRODUCTION

It has been suggested that it is unnecessary to apply the quantum theory to the gravitational field\(^{(1)}\) even though it interacts with quantum fields, if the gravitational field interacts with the expectation value of the energy tensor of these fields. It has also been proposed that one does not have to quantize the electromagnetic field to get the results of quantum electrodynamics\(^{(2)}\); e.g., the photoelectric effect has been consistently explained by Lamb and Scully\(^{(3)}\) within the framework of semiclassical radiation theory. There thus seems to be no compelling argument against the thesis that a classical field can in some manner interact with quantum particles and fields. We shall show, however, that the assumption that a classical field interacts with quantum systems in any physically reasonable fashion leads to violation of either momentum conservation, the uncertainty principle, or relativistic causality in the form of signals traveling faster than c.

Briefly, we show that if a gravitational wave of arbitrarily small momentum can be used to make a position measurement on a quantum particle, i.e., to "collapse the wave function into an eigenstate of position," then the uncertainty principle is violated. If the interaction does not result in collapse...
of the wave function, it is then possible to distinguish experimentally between superposition states and eigenstates. We show that this ability allows one to send observable signals faster than c when applied to a state consisting of two spatially separated particles with correlated spins.

2. MEASURING THE POSITION OF A QUANTUM PARTICLE WITH GRAVITATIONAL WAVES

The idea that the gravitational field is classical seems quite elementary, but we must specify precisely what we mean by this concept before we can consider how it might interact with a quantum system. First, for the gravitational field to be "classical," all of its components must simultaneously possess precise values. Second, the field must satisfy the usual wave equation, at least for weak amplitudes, i.e., the field possesses wave excitations or arbitrarily small amplitude and wavelength which may be superimposed to form wave packets. Further, the momentum and energy flux are proportional to the square of the amplitude. These statements imply that the position and momentum of a wave packet can be simultaneously well defined.

We now consider an experiment in which the position of a quantum particle is measured by scattering a classical gravitational wave from it. To utilize the classical gravitational field as a reliable probe of a quantum system, we must show that it is possible to determine the properties of the field (such as the momentum and localization of its wave excitations) to arbitrary precision, using only quantum systems as preparing equipment and as detectors. If this is impossible, then, at least on an operational level, it is doubtful if one even has a classical field, since the fact that it must be measured with quantized matter imposes an observational limit on the precision of the field variables. We analyze this question in detail in Appendix A, but briefly describe here how it is possible to perform such measurements.

Our method is to prepare and detect gravitational waves with quantum matter acting in the classical limit, so quantum uncertainties will result in negligible perturbation of the classical wave. While we wish our preparation and detection apparatus to operate in this limit, we desire the opposite limit to be in effect during the probing of a quantum system with a classical wave. That is, we wish the gravitational wave to leave the quantum system unperturbed. It is possible to accomplish these apparently conflicting aims. The fluctuations the quantum theory imposes on matter are inescapable, but the magnitudes of quantum parameters such as level spacings, binding energies, zero-point oscillations, etc., scale in a continuous and unrestricted fashion with the masses and coupling strengths of quantum systems. To