OPTIMUM RESIDUAL PLASTIC DEFORMATION DISTRIBUTION IN LOCAL STRENGTHENING OF THIN-WALLED STRUCTURES

V. N. Maksimovich

Local strengthening of the elements of thin-walled structures may be accomplished with the use of local heating [1-4]. This method of strengthening is based on the fact that as the result of heating residual plastic deformation occurs causing compressive stresses in the region of a stress raiser or in places with weakened strength.

This work considers problems of determining the optimum residual plastic deformation distribution of plates with stress raisers.

Let a plate occupy the region $D$ and on its boundary the stresses or displacements are specified. Let us assume that the residual plastic deformations $e_i^0 = e_i^0(x, y)$, $e_y^0$ and $e_{yy}^0$ are distributed in the region $S$ ($S \in D$). Then the stresses caused by the plastic deformations may be calculated using the equations

$$
\varepsilon_{ij}(x, y) = \int_S \left[ e_i^0(x_0, y_0) S_{ij}^0(x, y; x_0, y_0) + e_y^0 S_y^0 + e_{yy}^0 S_{yy}^0 \right] dx_0 dy_0,
$$

where $S_{ij}^0$, $S_y^0$, and $S_{yy}^0$ are the Green's function of the corresponding problems of the theory of elasticity. For example, the function $S^0_{ij}$ is determined as the stress $\sigma_{ij}$ for $e_i^0 = \delta(x-x_0)\delta(y-y_0)$ and $e_y^0 = e_{yy}^0 = 0$ with zero boundary conditions.

Let us be limited to the case of a single stress raiser and let us assume that its center point (for stress raisers small in dimensions) or the center point of the region of stress concentration (for large stress raisers) lies at the origin of the coordinates (point 0) and that its strength is determined by the stresses $\sigma_{yy}$ acting here. We will also accept the following assumptions: the main plastic residual deformations $e_1$ and $e_2$ at each point of the region $S$ satisfying the conditions

$$
e_i = k e_i, \quad f_i(x, y) \leq e_i \leq f_2(x, y),
$$

and the area of the region $S$ is a value specified in advance, that is,

$$
\int_S dx dy = |S| = \text{const.}
$$

Here $k$, $f_1$, and $f_2$ are the specified constant and functions. Taking into consideration conditions (2) the equation for determining $\sigma_{yy}(0, 0)$ may be represented in the form

$$
\sigma_{yy}(0, 0) = \int_S e(x_0, y_0) F(0, 0; x_0, y_0) dx_0 dy_0,
$$

where $F = (k \cos^2 \theta + \sin^2 \theta) S_{yy}^0 + (k \sin^2 \theta + \cos^2 \theta) S_x^0 + (k - 1) \sin 2\theta S_{yy}^0; \quad e = e_2; \quad \cos \theta$ and $-\sin \theta$ are the direction cosines of the main residual plastic deformations. From all of the regions $S$ and functions $e$ satisfying


© 1983 Plenum Publishing Corporation
conditions (2) and (3) it is required to find those for which the value of (4) would be negative and the largest possible in absolute value. From this we arrive at the determination of the extreme value of the functional

$$I = \int_{\mathbb{B}} e(x_0, y_0) F(0, 0; x_0, y_0) \, dx_0 \, dy_0$$

(5)

with conditions (2) and (3). By introduction of the Lagrange factor \( \lambda \) the given problem is reduced to establishment of the extreme of the functional

$$I_1 = \int_{\mathbb{B}} |e(x_0, y_0) F(0, 0; x_0, y_0) + \lambda| \, dx_0 \, dy_0,$$

with conditions (2) on the function \( e \). Since the functional \( I_1 \) is linear relative to the sought-for function, the extreme solution will be the following:

$$e(x, y) = f_j(x, y), \quad (x, y) \in S_j, \quad j = 1, 2,$$

where the region \( S_1 \) and \( S_2 \) are bounded by the curves \( f_j(x, y) F(0, 0; x, y) + \lambda = 0 \). Having been set by the different values of \( \lambda \), we obtain the set of boundaries and of regions \( S_1 = S_1(\lambda) \) and \( S_2 = S_2(\lambda) \) corresponding to them. We make a single-valued selection of \( \lambda \) and of the optimum region \( S \) of distribution of plastic deformations taking into consideration condition (3), that is, \( |S_1(\lambda)| + |S_2(\lambda)| = |S| \).

We should note that if the region \( S \) and the distribution of \( \epsilon_0, \epsilon_0^x, \epsilon_0^y, \) and \( \epsilon_0^{xy} \) are known, then such a problem will obviously be solved simultaneously. Among all of the possible residual plastic deformation distributions satisfying conditions (2) and causing the same compressive stresses, that with which the area of the region \( S \) is the least is determined.

Let us consider particular cases.

1. A large plate with a single dimensionally small stress raiser (such as a small crack directed parallel to the Ox axis with the center at the point \((0, 0)\)). Therefore we will assume the region \( D \) to be an infinite and continuous plate. The expressions for the functions \( S_{xy} \) with \( x \neq x_0 \) and \( y \neq y_0 \) are:

$$S_{xy} = \left( e^2 \right) \left[ \left( x^2 - y^2 \right) / 2r \right] + \left( y^2 \left( y^2 - 3x^2 \right) / r \right],$$

$$S_{xx} = \left( e^2 \right) \left( x, y \right) (x^2 - 3y^2) / r,$$

$$S_{yy} = \left( e^2 \right) \left( x^2 - y^2 \right) / r - \left( y^2 \left( y^2 - 3x^2 \right) / r \right].$$

Here \( x_0 = x - x_0, \ y_0 = y - y_0, \ r_0 = x^2 + y^2; \) and \( E \) is the modulus of elasticity. The functional \( I \) then may be represented in form (5) with

$$F = - \left( 1 / 4 \pi r \right)^2 \left( - (k + 1) \cos 2 \psi + (k - 1) \sin \psi \sin (\phi - 2 \psi) + 2 \cos 2 \psi \sin (\phi + 2 \psi) \right).$$

The solution with \( f_1 = 0 \) and \( f_2 = \text{const} \) will be the most simple. In this case the region \( S \) is the inner in relation to the curve \( L \) described by the equation \( F(0, 0; x, y) = l_1 \), and in the region \( S \) \( e = f_2 \). Having been set by the parameters \( k \) and \( \theta \), we obtain the set of curves \{L\} and areas \{S\}, which are dependent upon the value of \( l_1 \). In addition the area of the region \( S \) may be selected as such that the compressive stresses in the stress raiser reach the specified level.

Let us assume that \( \theta \) changes within limits of \( 0 \leq \theta < 2 \pi \). Then we determine \( \theta = \theta(x, y) \) so that the value of \( F(0, 0; x, y) \) as a function of \( \theta \) will be the minimum or the maximum. Obviously to obtain the same stresses in the stress raiser it is sufficient to treat the smallest region \( |S| \). It is not difficult to be convinced that the largest and smallest values of the function \( F \)

$$F = - \left( 1 / 4 \pi r \right)^2 \left( - (k + 1) \cos 2 \psi + 2 (k - 1) \sin \psi \sin (1 + 2 \cos^2 \psi) \sin (1 - 2 \cos \psi) \right)$$

are provided with angles \( \theta \) satisfying the equations \( \cotan \theta = \tan \theta (1 + 2 \cos 2 \phi) / (2 \cos 2 \psi - 1) \).

Let us dwell in more detail on the case of \( k = 1 \). The equation of the contour \( L \) has the form \( \cos 2 \phi = C \gamma^2 \), and with \( \epsilon < 0 \) for obtaining compressive stresses in the stress raiser the angular variable changes within limits of \( |\phi| < \pi / 4 \) and \( |\phi - \pi| < \pi / 4 \).

The strengthening is done in such a manner that the stress raiser does not fall within the region of plastic deformations [4]. Let us determine the compressive stresses at the point \( O \) if the region \( S \) is bound by the curve \( \rho_0 \cos 2 \phi = \gamma^2 \) and a portion of the circumference \( r = \rho_0 \) \( (\rho_0 < \rho_1) \). Here \( \rho_0 \) is the distance of the region of plastic deformations from the stress raiser. By integration we find

$$\sigma_{yy} (0, 0) = (Ee / 2n) \left[ \ln ((1 + \gamma^2 - \gamma^2) / (1 - \gamma^2)) - \gamma / (1 - \gamma^2) \right],$$

$$\gamma = \rho_0 / \rho_1^2. \] The area of the region \( |S| = \rho_1^{(2)} \left( 1 - \gamma^2 / \gamma \right) \] arccos \( \gamma \). We calculate the constant \( e \) so that the residual