NUCLEATION AND GROWTH OF A MACROCRACK IN TENSION AND COMPRESSION

V. P. Naumenko

INTRODUCTION

The cracking resistance and residual strength of structures made of brittle materials, deformed under static loading in tension and compression, are usually determined on the basis of different approaches and separately from each other. Like previously, the methodology of examining failure under compression is based at present mainly on the assumptions of mechanics of continuum and conventional (macroscopic) strength conditions. The criteria and methods of linear fracture mechanics are used extensively and successfully in the problems of tension (bending). This discreteness in examination of the given phenomenon, which appears unnatural, is partially caused by large differences between the relationships governing the development of the failure process in tension and compression. For example, homogeneous fields of compressive stresses often contain long areas of simultaneous growth in parallel planes of many separation cracks scattered in an ordered manner throughout the entire deformed volume of the material. Conversely, in tensile loading these regions are not found or the number is very small. In addition, they are comparatively small and, in most cases, localized in the zone with the highest stress concentration.

The discreteness in approaches to examining brittle failure in compression and tension started in fundamental studies by Griffith [1, 2]. A number of attempts have been made to overcome it [3, 5]. According to the current state, these attempts have not yielded the required result. In this article, we briefly describe a general approach to solving two relatively simple and important problems relating to failure in tension and compression. This concerns the cases of nucleation and growth of continuous separation macrocracks from the surface of a circular hole in sheet or shell structures. These cases occur very often in practice. A successful solution may be found by using in the examined approach the generalize model (p-model) of brittle failure [5] based on the hypothesis on the unique nature of the phenomena of failure by separation in compression and tension.

STRESS INTENSITY FACTORS

We shall examine statically loaded in tension and compression unlimited linearly elastic plates weakened with a circular hole and two symmetric radial cracks of the same length (Fig. 1). Our aim is to determine stress and strain fields in the vicinity of crack tips. For both problems, these fields, characterized by the stress intensity factors $K_i$, were determined by many investigators. The corresponding solutions are based on the assumption according to which the three-dimensional nature of the crack cavity in the real material as well as the residual stresses, maintaining this crack in the open form in the absence of external loading, can always be ignored. Therefore, in calculations instead of the crack we examine a mathematical section with edges free from loading. The length of this section is equal to that of the crack. Previously, it was shown [5, 6] that these assumptions are unacceptable, at least in analysis of loading cases along the crack line.

Taking these assumptions into account, we shall use a solution of an auxiliary problem of biaxial loading of an unlimited linear elastic plate by tension $\sigma$ along the normal and compression $q = k\sigma$ along the line of an isolated continuous crack with the length $2a$ [5]. The parameter $K_i$ for this configuration is calculated from the equation

$$K_i(k) = \sigma [\pi t(k)]^{1/2},$$

(1)
Fig. 1. Uniaxially loaded plates with continuous radial cracks at the contour of a circular hole
a) compression; b) tension; c) diagram of macrocrack initiation.

where $k = q/a$ is the parameter of biaxial loading; $l(k)$ is the half length of the section simulating the crack:

$$l(k) = a + 0.5 \rho k^2 + 0.375 (\rho a)^{1/2} (1 - k)^2,$$

$\rho$ is the linear dimension typical of the separation macrocrack in this material. The value of $\rho$ is determined from the results of two experiments carried out at different biaxiality parameters $k$ [5, 7].

Superposition of Eq. (1) and the solutions of the problems of two identical radial sections at the contours of an isolated circular hole [8, 9] gives the following relations:

$$K_1 (-\infty) = q \left\{ \frac{l(-\infty) + R}{l(\infty) + R} \right\}^{1/2} F_q(\lambda),$$

in compression, where $k = -\infty$, and

$$K_1 (0) = \sigma \left\{ \frac{l(0) + R}{l(0) - R} \right\}^{1/2} F_q(\lambda),$$

in tension ($k = 0$), where

$$l(-\infty) = 0.5 \rho + 0.375 [\rho (a - R)]^{1/2},$$

$$l(0) = (a - R) + 0.375 [\rho (a - R)]^{1/2}.$$  

The functions of the effect of the hole $F(\lambda)$ have the form

$$F_q(\lambda) = \left\{ \lambda_q (1 + \lambda_q) \right\}^{1/2} \left\{ (2/\pi) (1 + \lambda_q)^{-4} [(2\lambda_q + 3\lambda_q^2 + \lambda_q^3) (1 + \lambda_q)^{-1} \right\}^{1/2};$$

$$F_*(\lambda) = \left\{ \lambda_q (1 + \lambda_q) \right\}^{1/2} \left\{ 1 + 0.358 (1 + \lambda_q)^{-1} + 1.425 (1 + \lambda_q)^{-2} - 1.578 (1 + \lambda_q)^{-3} + 2.156 (1 + \lambda_q)^{-4} \right\}.$$  

where

$$\lambda_q = e(-\infty)/R \quad \text{and} \quad \lambda_0 = e(0)/R.$$  

The functions (7) and (8) are in satisfactory agreement with the available analogs and, at the same time, are relatively simple and easy to use. According to definition, the function $F_q(\lambda)$ coincides with the solution taken from [9], on the condition that $\rho/R = 0$. With an increase of $\rho/R$ its maximum value ($F_{\text{max}}^q = 1.057$), corresponding to the section $l(0) = 1.52 R$ at $\rho/R = 0$ (Fig. 2), is displaced and, for example, at $\rho/R = 0.0205$ and 0.205 occupies consecutively the position $l(0)/R = 1.45$ and 1.33. As expected, the problem of compression contains qualitative differences between the traditional and new calculation methods. In the former case, the function $F_q(\lambda)$, which is nonmonotonic with the maximum ($F_{\text{max}}^q = 0.206$) at $l(-\infty)/R = 1.18$, tends asymptotically to zero [8], whereas in the latter case it is monotonic and tends to unity.