The theory of continuously operated chemical reactors to which part of the reaction mixture leaving the reaction zone is recycled and which are finding ever wider application in chemical and petrochemical processes, was developed in papers [1-5]. The authors derived analytical equations describing the statics (macro-kinetics) of the various important irreversible, reversible, and autocatalytic processes in the reactors. Analysis of the results enabled us to find interesting properties of recycled chemical reactors.

In particular, it was established that in many cases (simple irreversible first-order reactions etc.), the capacity of flow reactors to which the product is recycled is intermediate between the capacities of ideally-mixed and ideal prop flow reactors. It was further established that, if autocatalytic first-order reactions are carried out in such reactors, there may be found an optimum degree of recycling at which the capacity of the recycled reactor exceeds that of ideally-mixed and ideal prop flow reactors.

Below we report a graphical method for calculation of recycled reactors. A typical feature by which the graphical method differs from the analytical calculation method is that with the former one has not to use an analytical expression for the kinetic plot from the experimental data. The method is simple and graphical. It enables the optimum degree of recycling to be rapidly calculated with a minimum amount of computation work. It contributes to a better understanding of the specific operation of a recycled reactor, permits evaluating the position of this reactor type relative to that of ideally-mixed and ideal prop flow reactors from the shape of the original kinetic plot.

We shall discuss the very general case that a simple kinetic reaction takes place in the apparatus. Such a process can be described by a single differential equation of the type:

$$\frac{dx}{dt} = \Phi(x), \quad (1)$$

where $x$ denotes the concentration of one of the reaction products (key component); $\Phi(x)$ is the kinetic function determined by the reaction type.

Figure 1 shows a diagram of a flow reactor to which the products are recycled. We shall use the following simplifying assumptions: the reaction takes place without causing a change in volume, no radial velocity gradient exists in the apparatus, the hydrodynamical regime of ideal prop flow establishes, and the system is operated isothermally. The mass balance taken over the total current and the reaction product in the point where the fresh current is mixed with the recycled stream then reads:

$$\Phi_{vmf} = \Phi_v + \Phi_{vr} \quad (2)$$

$$\Phi_{vmf} x_{mf} = \Phi_v x_n + \Phi_{vr} x_k \quad (3)$$

where $\Phi_v$ denotes the volume of fresh material supplied per unit time to the reactor, $\Phi_{vr}$ the flow rate of the recycled current, $\Phi_{vmf}$ the total flow rate (after mixing of the fresh and recycled currents) of the current entering the reactors; $x_n, x_k, x_{mf}$ denote the concentrations of the key component in the fresh, recycled, and total streams, respectively.

Introducing the notation $r = \Phi_{vr}/\Phi_v$, where $r$ denotes the degree of recycling, we derive

$$\Phi_{vmf} = \Phi_v (1 + r), \quad (4)$$

$$x_{mf} = x_k - (x_k - x_n)/(1 + r). \quad (5)$$

The principles of the derivation of macrokinetic relationships for recycled reactors are discussed in more detail in papers [2, 5].

The mean residence time of the reactant liquid $\tau$ in the apparatus equals

$$\tau = \frac{O_p}{\varphi} = \frac{O_p}{\varphi_{vmf}} (1 + r) = \tau_{mf}(1 + r), \quad (6)$$

where $O_p$ denotes the effective reactor volume, $\tau_{mf}$ the residence time of the mixed flow in the apparatus (the recycled part being taken into account). To find $\tau_{mf}$, we return to the kinetic Eq. (1). We shall introduce the parameter

$$\frac{dx}{dx} = \frac{1}{\psi(x)} = \psi(x), \quad (7)$$

which may be considered as the differential time increment needed for increasing the conversion from the concentration $x$ to $x + dx$.

The residence time of the flow $\tau_{vmf}$ in the apparatus then equals

$$\tau_{vmf} = \int_{x_{mf}}^{x_k} \psi(x) \, dx,$$

hence it follows that

$$\tau = (1 + r) \int_{x_{mf}}^{x_k} \psi(x) \, dx.$$

We now assume that the plot of the differential time $\psi$ versus $x$ is known (Fig. 2).

The differential reaction time $\psi = d\tau/dx$, which is the reverse of the true reaction rate, has quite often been used in graphical kinetic calculations by Hougen and Watson and other authors. Plots of $\psi$ versus $x$ can be constructed in two ways. First, by graphical differentiation of experimental $x-\tau$ plots taken either in a batch reactor or in a continuously operated ideal prop flow reactor. Second, by starting from $x-\theta$ plots taken in an ideally-mixed reactor to which a current without the key component (reaction product) is supplied ($\theta$ denotes the mean nominal residence time). In the latter case the plot needed is constructed by applying the relationship $\theta/x = d\tau/dx = \psi(x)$.

Let the points $x_n$ and $x_k$ on the axis of abscissas (Fig. 2) represent the concentrations of the key component at the inlet and outlet of the reactor system. To find the position of the point $x_{mf}$, we divide the line segment $x_n x_k$ in the ratio $r : 1$, so that $x_n a : x_k a = r$. The length of the segment $a x_k$ will then be equal to $r + 1$ times the length of the segment $x_n x_k$, i.e.,

$$\frac{ax_k}{x_n x_k} = \frac{ax_k}{x_n x + dx_k} = \frac{1}{1 + r}.$$  

Consequently, the abscissa of point $a$, or the length of segment $oa$, equals

$$oa = \frac{x_n x_k}{1 + r} = \frac{x_k - x_n}{1 + r} = x_k - x_n = \frac{x_k - x_n}{1 + r}.$$  

Hence, it follows that point $a$ represents the concentration $x_{mf}$.

We shall now graphically determine the mean value of function $\psi(x)$ in the interval $x_{mf} - x_k$. The mean value of function $\psi(x)$ in the interval $x_{mf} - x_k$ equals